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Zoology. — "*Physiological regeneration of neurofibrillar endnets (tactile discs) in the organ of Eimer in the mole.* By Dr. J. BOEKE and Dr. G. J. DE GROOT. (Communicated by Prof. G. C. J. VOSMAER).

(Communicated in the Meeting of November 30, 1907).

In recent years several authors (RANVIER, VON LENHOSSEK inter alia) have called attention to the fact, that there where intraepithelial nerves in mucous membranes or in the epidermis are found penetrating even between the superficial layers of epithelium cells covering the sensory surface (so for example in the peribulbar nerve-endings between the taste-buds in the papillae of the tongue, etc.), we have to draw the conclusion, that at the same time as the superficial cells degenerate and are cast off, the sensory nerves with their knob-like end-swellings or end-loops of the neurofibrillae must undergo a perpetual change and growth. But then these are always the fine ramifications and endings of the nerves, which branch between the deeper layers of epithelium cells. Real neurofibrillar endnets like those which are formed round the base of the tactile cells of MERKEL, are always found in the deeper layers of the epidermis, where they lie protected by the other epithelium cells. These tactile cells nowhere degenerate so quickly as it is the case with the superficial cells of the upper layer of the epithelium, and need not be replaced by other cells coming from the deeper strata. There is no need of a quick regeneration of the neurofibrillar endnets (and the tactile cells).

But suppose we had a tissue, where in the uppermost strata of a stratified epithelium, in which the superficial cells quickly degenerate and are cast off, we find real tactile cells with distinct neurofibrillar endnets, which therefore must degenerate at the same time as the surrounding cells, how would the process of regeneration of the neurofibrillae take place there?

In the course of investigations carried on in the histological laboratory at Leiden we found a favourable object to study this question in the sensory organs in the snout of the earth-mole (Talpa europaea).

Here we find an extremely sensitive tissue (the organ of EIMER) the elements of which are only protected by a very thin horny layer, and which by reason of its lying at the tip of the snout must, on account of the well-known habits of the animal, continuously form new horny cells for the protecting horny layer above, because otherwise the functional cells would very soon come to lie at the surface and be liable to be injured.

The structure of the peculiar organ first described by EIMER (1870) and the innervation of it, have been studied in the course of this year (1907) by two authors [1]) by means of the recent improved methods of staining the neurofibrillae. Both give about the same description but arrive at different conclusions.

As is well known, the organ of EIMER consists of thickenings of the epidermis, formed by columns of epithelial cells in the shape of an hourglass, which form small round prominences on the surface of the snout, and which, because the columns of cells are longer than the thickness of the epidermis at the place where they are found, project with their base into the corium, and form here a bulging out of the epithelium, generally described as "buffershaped". Each of the columns is made up of several strata of more or less flattened epithelial cells, which at the base of the column do not reach from one side to the other, but are wedgeshaped and overlapping each other with the thinned-out ends. Nearer the surface the cells gradually become flattened and larger, until only two cells lying at the same niveau, fill out the entire cross-section of the sensory column (fig. 1, 5). There the column ends as it reaches the horny layer. The cells of the column are, according to BOTEZAT, true spiny cells like the other cells of the epidermis (fig. 3).

In the axis of the column a thick nerve fibre, the axial fibre, runs through the whole length of it, penetrating into the epithelium at its base. Sometimes there are two or three axial fibres. Around the column of cells a set of 18 or 19 thin, unbranched nerve fibres, closely set, somewhat zigzag, run upwards between the outer ends of the cells of the column and the adjoining epidermis-cells, until they reach the horny layer. These are called rand-fibres to distinguish them from the axial fibre. At the base of the column between the epidermis-cells a small number of tactile cells of MERKEL are found, and underneath the epidermis in the corium one or two small Pacinian corpuscles.

EIMER already described small varicosities or knoblike swellings of the nerve-fibres in the upper part of the columns. The nerve-fibres run more or less zigzag between the cells. EIMER himself and after him HUSS (1898) thought that these knoblike varicosities were lying intracellular, the nerve-fibres running between the cells. The varicosities are therefore attached laterally to the nerve-fibres.

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1) EUGEN BOTEZAT. Anat. Anzeiger, 30 Bd. 1907.

M. BIELSCHOWSKY. Anat. Anzeiger, 31 Bd. 1907.

In this year (1907) BIELSCHOWSKY [1]) has investigated the nerves of the organ of EIMER by means of his method of staining the neurofibrillae, and although he does not give much that is new, as he says himself in his paper, his study is interesting because with that by BOTEZAT it is the only one, in which the new staining methods for the nervous system are used for this organ. We may quote here what he says about the course and the peculiarities of the nerve-fibres, because this makes clear his opinion better than a long description. The course of both the axial fibre and the randfibres he assumes to be entirely intercellular: "irgend ein näherer Konnex der Fasern zu den Epithelzellen findet nicht statt; ihr Verlauf ist ein rein intercellulärer. Im Bereiche der äusseren Schicht weisen sie in scheinbar regelmässigen Abständen die bekannten punktförmigen Varikositäten auf..... Die Varikositäten sind offenbar nur auf Zerfallsvorgänge zurückzuführen. Dafür spricht der Umstand, dass sie immer erst in der Verhornungszône des Epithels deutlich hervortreten. Aehnliche Beobachtungen kann man auch am Schweinerüssel and anderen rüsselformigen Säugerschnauzen machen". (l.c. p. 189).

In his last paper, published some months before the paper by BIELSCHOWSKY appeared (April 1907), BOTEZAT [2]) who in his paper of 1903 pronounced the same opinion as EIMER and HUSS, viz. that the knoblike thickenings of the nerve fibres penetrate into the cells of the column of EIMER, adopts the view that they are epicellular, after a study of the nerves coloured with methylene blue and after the method of RAMON Y CAJAL. "Der Beweis hierfür lässt sich am besten dadurch erbringen, dass man die Terminalknöpfchen fast genau zwischen den Zellen des Organs liegen sieht." BOTEZAT states that the varicosities possess a netlike structure. Because they are excessively small, the extreme sensibility of the snout must be due to the very large number of the terminal knobs ("tactile discs") and not to their great perceptibility. A column of EIMER consists of about 15 layers of cells, and in each layer about 20 of these tactile knobs are to be found. The total number therefore is in each organ of EIMER 300, and for the entire snout more than 100000 [3]). According

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[1]) M. BIELSCHOWSKY, Ueber sensibele Nervenendigungen in der Haut zweier Insectivoren (Talpa europaea und Centetes ecaudatus). Anat. Anzeiger. Bd. 31, p. 187—194, 1907.

[2]) EUGEN BOTEZAT. Ueber die epidermoidalen Tastapparate in der Schnauze des Maulwurfs etc. Archiv für Mikroskopische Anatomie. Bd. 61. p. 730—764. 1903.

[3]) EUGEN BOTEZAT. Die fibrilläre Struktur von Nervenendapparaten in Hautgebilden. Anat. Anzeiger. Bd. 30. p. 321—344. 1907.

to BIELSCHOWSKY the total number is ± 150,000, together with more than 5000 end-bulbs and numerous cells of MERKEL.

Although he does not ascribe to the varicosities a high degree of perceptibility, BOTEZAT assumes all of them (both of the axial fibre and of the rand-fibres) to be tactile discs, in accordance with most authors. A difference in structure between the different tactile discs or knobs he mentions without paying much attention to it.

Now the facts seem to us to point to a different conclusion.

The opinion of BIELSCHOWSKY, that the varicosities of the nerve-fibres are due to "Zerfallsvorgänge", seems to us to be erroneous. In the first place these varicosities do not appear first in the horny zone. On the contrary, as soon as the cells are transformed into horny cells, the fibres and their varicosities degenerate, and the first varicosities appear seven to eight layers of cells lower down. In the second place the varicosities are much too regular and are distributed with a far too great regularity to be the mark of degeneration, and are always present in nearly the same number. In the third place their structure does not point at all to "Zerfallsvorgänge."

But in his description BOTEZAT too does not seem to have hit the point. He does not give an explanation of the difference in structure of the varicosities and of their mode of attachment to the nerve-fibres, and of the fact that they are only to be found in the peripheral part of the nerve-fibres and not in the basal half.

When we treat a small piece of the snout of the mole, after fixation in formaline, according to the method of BIELSCHOWSKY—POLLACK, and study a correctly differentiated preparation in thin (6 μ) longitudinal sections (that is a longitudinal section of the nerve-fibres and of the column of cells, the section being made at right angles to the surface of the epidermis of the snout), the following details will be seen: the structure and form of the varicosities ("Terminalknöpfchen, Seitenknöpfchen") are not the same in the course of the nerve-fibres. When we follow a rand-fibre from the base of a column of EIMER to the top, the first swellings appear at a distance of 10 to 12 cell-layers from the top (fig. 1, 5). The swellings are here only loosely built small nets, lying in the course of the nerve-fibres, nothing but a local slackening of the bundle of neurofibrillae in the fibres, the fibrillae probably forming a few anastomoses. From this point upwards we see these networks appearing with great regularity in the course of the nerve-fibres where the fibre passes another cell of the column, and each time the reticular structure becomes finer and more distinct (fig. 1, 3).

In the upper four to five rows of cells a change in the form and

arrangement of the networks becomes visible. The small swellings of the nerve-fibre no more lie in the course of the nerve-fibres, but more and more pass to the side of it (fig. 1, 3) and at last they lie entirely beside the nerve-fibre, being connected with it by means of a very small and short stalk (fig. 1, 2, 3). The swellings of the rand-fibres always pass to that side of the fibre lying close against the cells of the column of EIMER, and so project centripetally (fig. 1, 6). So when we look at a rand-fibre from the outside of the column, as in fig. 5, we see nothing or only very little of this change of place of the varicosities, and only when we play up and down with the micrometer-screw of the microscope, we are able to make out that the peripheral rows of varicosities lie in reality underneath the fibres.

So in the first place we see a very regularly occurring change of place of the varicosities, as the fibre approaches the surface of the epithelium. When we only take the place of the fibre we are examining in the section into account, this change is always found to take place with perfect regularity.

In the second place the following change may be seen: the nerve-fibres of the organ of EIMER (both the rand-fibres and the axial fibres) run between the cells of the epithelium. The first swellings or varicosities, the small loose nets lying in the course of the fibres, of course also appear between the cells. But as soon as these varicosities get larger and change their places, so that they come to lie besides the fibres, they push their way *into* the substance of the cells of the sensory column and not between these cells. They become *intracellular*. In the preparations stained after the method of BIELSCHOWSKY the cells and their margins and nuclei are so clear and distinct, that when we only take care to examine thin sections (5—6 μ), this fact may be stated with perfect clearness. Fig. 1, 2 and 3 give a good idea of it; when we examine longitudinal sections of the rand-fibres, the section passing through the axis of the sensory column, we see as it were the varicosities or knobs push their way into the protoplasm of the cells. In cross-sections now and then we come across places, one of which is figured in fig. 4. The tactile knob growing into the flat epithelial cell, pushes its way into the protoplasm apparently with some force so that the flattened nucleus is curved in by it. Similar drawings are given by HUSS.

Another question is, whether these varicosities or tactile knobs lie in the protoplasm of the cell, become an integrating part of it. The facts seem to point to the contrary. On observing

the fibres and their tactile knobs closely under the highest power, we get the impression that even there where the knobs lie intracellularly, the neurofibrillae are still surrounded by a very thin layer of perifibrillar substance, taking a different stain from the protoplasm of the cell itself. But of course this layer of perifibrillar substance must be continuous with the surrounding protoplasm. The neurofibrillar network remains entirely independent, but a trophic connection of the perifibrillar substance and the protoplasm surely must be present. This seems to us to be beyond doubt, and we may venture to suggest, that only now the varicosities reach their full development, are real tactile discs; as long as they lie between the cells, the varicosities are only parts of the nerve-fibres where the neurofibrillae are getting looser and growing out, but only when they pass to the side of the fibres and grow into the cells, they become real tactile neurofibrillar end-nets. The rows of varicosities are merely stages of development of the tactile discs.

The end-knobs or terminal discs in the upper row of cells of the sensory column, which are already on the point of passing into the horny layer, are for the greater part already lying loose in the cells, the nerve-fibres themselves and the connecting stalks atrophying. So in fig. 5 the four knobs, represented by black spots in the upper row of cells, are entirely separated from the nerve-fibres below, and the same fact is to be seen in the fig. 1 and 3, where a part of the nerve-fibre (the stalk of the end-knob) was still stained. The argument, that this independence of the terminal knobs is due to the connecting stalks not being cut in the section examined, is annihilated by a close study of many sections. Thus we can state with perfect accuracy, that the connecting fibre really does not exist any more (at least, is not stained as the functional fibres are).

The axial fibre shows the same peculiarities as the rand-fibres, but the tactile nets are larger and more rounded; the axial fibre too runs between the cells until its end; even there where, in the upper part of the column, the entire cross-section is composed of two cells, the line between these cells runs just through the middle of the transverse plane (cf. Huss) and leaves a small room just in the axis of the column, occupied by the axial fibre (fig. 6). The tactile nets grow out from the fibre now at one side and then at the other, and grow into the cells of the sensory column just as it was described for the rand-fibres.

So we find the same peculiarities of structure in all the nerve-fibres and their tactile neurofibrillar networks. The same cause seems to us to underlie all these differentiations, which we may describe

as a getting more and more differentiated and independent of the tactile discs (or varicosities) as we draw nearer to the surface of the sensory column.

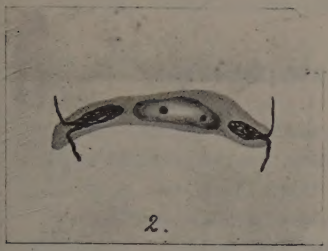
When we see now, that the horny layer above the columns of the organ of EIMER is always thinner than in the adjacent parts of the epidermis (so for example in fig. 1, st. c.), as it was the case in all the preparations examined, and when we bear in mind, that these prominences on the surface of the snout of the mole are continually exposed to all sorts of mechanical insults, the question, put at the beginning of this paper, may be answered in the following manner:

The horny layer above the cells of the column of EIMER being very thin and composed of a few layers of cells, and the horny scales being lost very soon by desquamation, there must be a continual moving upwards of the cells of the deeper layers of the column of EIMER, to take the place of the thrown off cells. With these cells the nerve-fibres must grow upwards at the same rate. About in the middle of their course these nerve-fibres begin to form tactile discs. These corpuscles first appear as places in the course of the fibres where the neurofibrillar structure is looser; these first varicosities little by little pass out of the course of the fibre and grow into the cells of the column of EIMER, and so become real tactile discs. These tactile discs lying at the side of the nerve-fibres and remaining attached to them by a short stalk, are a direct argument for the growing upwards of the nerve-fibres together with the cells of the column. Otherwise the cells would take with them the tactile corpuscles and sever them from the fibres they belong to or draw out the stalks in an oblique direction. Of this no trace is to be found anywhere. It is very probable, that only when the varicosities grow out to small tactile discs and come to lie intracellularly, they acquire a heightened perceptive faculty. As they are continually travelling upwards to the surface, new varicosities are formed underneath in the course of the same nerve-fibre. As soon as the cells undergo the transformation into horny scales, the tactile discs and their connecting stalks and the nerve-fibres atrophy, the former remaining visible longer than the latter. Only the upper rows of tactile discs, of the form of the networks of fig. 2 and fig. 3, seem to be fully developed.

They are continually replaced by others, coming from below. The nerve-fibres of the column of EIMER chiefly grow at the base of the column and atrophy at its top.

Perhaps these views may be extended to other intraepithelial

. BOEKE and G. J. DE GROOT, "Physiological regeneration of neurofibrillar endnets (tactile discs) in the organ of Eimer in the mole."



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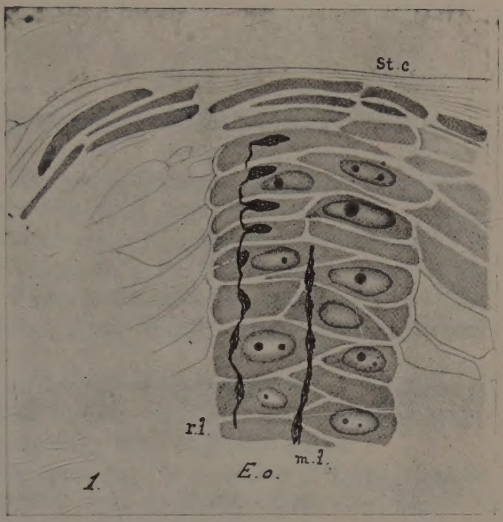

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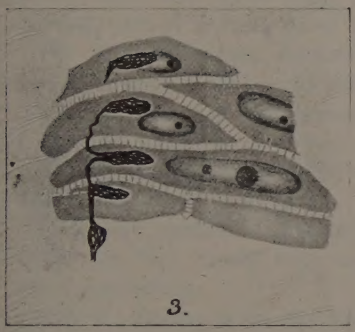

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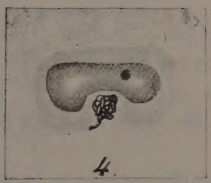

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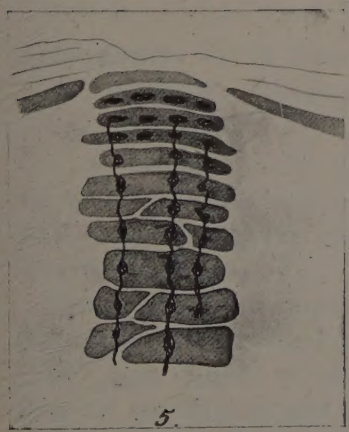

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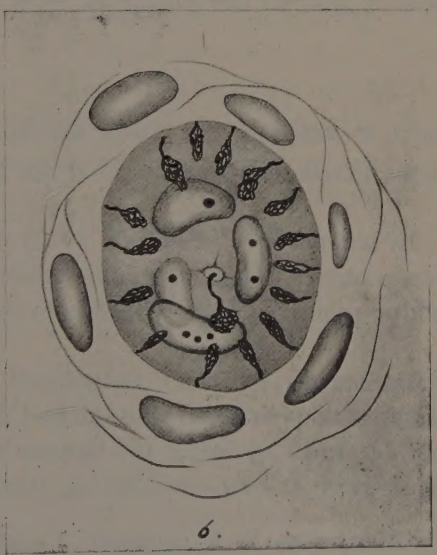

6.

J. Boeke a. n. del.

nerve-endings. It will be difficult to find an object of study as favourable as the organ of EIMER.

Leiden, Anatomical Cabinet.

DESCRIPTION OF THE FIGURES ON THE PLATE.

All the figures are drawn from life from preparations made after the method of BIELSCHOWSKY-POLLACK, with a camera lucida of ABBE. Fig. 1 and 5 are enlarged 1200 times, the others 1600 times. Apochromate-oil-immersion. Sections 5 and 6 μ.

Fig 1. Longitudinal section of the upper part of a column of EIMER of the earth-mole. A rand fibre (*rf*) and a part of an axial fibre (*mf*) are seen. The horny layer (*stc*) above the column of EIMER is distinctly thinner than at both sides of it.

Fig. 2. Longitudinal section of a flat cell of the upper part of a column of EIMER, with two tactile discs, growing into the same cell. The netlike structure and the curious drawing in of the connecting fibre, is clearly shown.

Fig. 3. Longitudinal section of the upper part of a column of EIMER, to show the developing of the tactile discs, and the final atrophying of the nerve-fibre.

Fig. 4. From a cross-section through the upper part of a column of EIMER. A nucleus curved in by a tactile disc.

Fig. 5. Longitudinal section through the peripheral part of a column of EIMER. Three rand-fibres are shown. The tactile discs lie behind the nerve-fibres. The intracellular position of the tactile discs is clearly to be seen. The upper cell, in which lie four tactile discs, is being transformed into a horny cell. The nerve-fibres degenerate.

Fig. 6. Cross-section through the upper cells of a column of EIMER. In the section of 6 μ four cells were to be seen, lying two and two in the same niveau. The tactile discs of the rand-fibres all grow centripetally into the cells, the axial fibre runs between the cells.

Astronomy. — "*β Lyrae as a double star.*" By J. STEIN, S. J. at Rome. (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

1. As far as I know, Professor E. C. PICKERING was the first who, led by his spectroscopic investigations, suggested that β Lyrae might be a close double, the components of which describe circular orbits in a light-period [1]).

This surmise was confirmed by BELOPOLSKY [2]) in 1892. He measured the displacement of the luminous *F*-line on some fourteen spectographs. They were found to show a minimum (in absolute value) at the time of the minima and a maximum at the time of the maxima of

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[1]) Spectrum of β Lyrae. By Prof EDWARD C. PICKERING. A. N. 3051 (1891).

[2]) Les changements dans le spectre de β Lyrae. A. BÉLOPOLSKY. Memorie della Società degli Spettroscopisti Italiani. Vol. XXII, 1893.

the star's light, in such a way that they correspond to an approach before the principal minimum and to a recession after that time. From these observations he derived a circular orbit for that component which eclipses the other at the time of this minimum. The investigation of the Potsdam spectographs equally led Prof. VOGEL [1]) to the conclusion that the displacement of the lines can hardly be explained otherwise than as a consequence of the motion of different bodies having unequal spectra. He does not succeed however in determining the position of the lines with sufficient accuracy. He thinks that the photometric data would lead to the assumption of two bodies of unequal luminosity moving either in a fairly circular orbit or in an ellipse having its major axis in the visual line. On the other hand the spectroscopic investigations would lead to the assumption of two bodies, one showing a spectrum with luminous, the other a spectrum with absorption-lines, which would describe very excentric orbits the major axes of which would make a considerable angle with the visual line. It would be impossible, in his opinion, to satisfy the two phenomena at the same time. In 1896 Dr. MYERS [2]) subjected ARGELANDER's lightcurve ("vera" pro 1850) to an elaborate theoretical investigation. His result is that the whole curve of the lightvariation is represented satisfactorily by assuming two elongated revolution ellipsoids the major axes of which are in each other's prolongation, circulating around each other in nearly circular orbits.

The next year BELOPOLSKY found the duplicity confirmed [3]). This time it was the displacements of the dark *Mg*-line ($\lambda = 448.2\ \mu\mu$), which enabled him to derive a slightly excentric orbit for the second component viz. of that component which is eclipsed during the principal minimum. Father W. SIDGREAVES, in his latest spectrographic investigation of β Lyrae [4]) arrives at the same result as Prof. VOGEL: rather considerably excentric orbit, the major axis of which makes a great angle with the visual line.

In conformity with what had already been suggested before by

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[1]) Ueber das Spectrum von β Lyrae. Von H. C. VOGEL. Sitzungsberichte der K. Preussischen Ak. der Wiss. zu Berlin. 8 Februar 1894.

[2]) Untersuchungen über den Lichtwechsel des Sternes β Lyrae. Inauguraldissertation... von G. W MYERS, München 1896. — The system of β Lyrae. id. The Astroph. Journ. Vol. VII N°. 1.

[3]) Recherches nouvelles du spectre de β Lyrae, par A. BÉLOPOLSKY. Memorie della Società degli Spettrosc. It. vol. XXVI, 1897. — New Investigations of the Spectrum of β Lyrae, id Astroph. J. Vol. VI N°. 4.

[4]) A spectrographic Study of β Lyrae. By Rev. WALTER SIDGREAVES S. J. Monthly Notices of R. A. S., Jan. 1904.

Dr. MYERS, Prof. CH. ANDRÉ [1]), basing himself on different numerical data, thinks himself justified in assuming, that the excentricity of the orbit has increased since the time of ARGELANDER, and also that the major axis has been displaced. On this supposition ANDRÉ tries to found an explanation of the terms of a higher order in the formula of ARGELANDER as corrected by Dr. PANNEKOEK [2]). Finally Dr. L. TERKAN has brought forward some short considerations in A. N. n° 4067 [3]). Afterwards a more elaborate investigation has appeared in the Memoirs of the Hungarian Academy of Sciences [4]).

We think that this enumeration covers the principal literature about what has been put forward in *explanation* of the light-variation.

2. The original plan of the author of the present paper was a treatment by the method of MYERS of the light curve derived by Dr. PANNEKOEK, in order to ascertain whether any important change of the elements of the orbits since the time of ARGELANDER, might be established.

The first part of MYERS' thesis in which, as a first approximation, a circular orbit is derived, is generally fairly correct. But the second part in which this orbit is changed to a slightly excentric one, by the aid of differential formulae, appeared to call urgently for a fresh treatment. Erroneous normal equations have been derived from incorrect differential formulae. The former have been wrongly solved and finally the close adjustment of the theoretical curve to that of ARGELANDER, chiefly in the vicinity of the principal minimum, seems to have been obtained by a happy coincidence of numerical errors. It is of no use to enter into further particulars on the subject. As an instance we give in the 2nd column of the following table the light-intensities (I_B), as derived by MYERS from the observed grades (Stufen) of ARGELANDER during the period of from 30 hours before to 30 hours after the principal minimum. In the next column are contained the light-intensities (I_{C_1}) given by MYERS as resulting from the definitive elements of his orbit [5]), the 4th col. shows these same quantities freed from numerical errors. In the three last columns the

[1]) Traité d'Astronomie Stellaire par CH. ANDRÉ, 2me p. NN. 460—1.

[2]) Untersuchungen über den Lichtwechsel von β Lyrae. Dr. A. PANNEKOEK. Verhandelingen der Kon. Ak. van Wetensch. te Amsterdam, Vol. 5, N°. 7. id. A. N. N°. 3456.

[3]) Beitrag zur Berechnung dər Bahnelemente van β Lyrae. Dr L. TERKÁN.

[4]) β Lyrae pályaelemeinek kiszámítása spektroskopikai és photometriai adatokból. TERKAN Lajostól. — Mathematikai és Természettudományi Ertesitö, XXIV·kötet 3 füzetéböl Budapest 1906.

[5]) Inaugural-dissertation, p. 48; A. J. l.c. p. 16.

same quantities have been given reduced to light-grades (σ). I do not find mentioned what is the value of a light-grade of ARGELANDER according to MYERS. From the light-intensities in the two minima I find 0.130 magnitudes, a value to which I have adhered. The intensity of the maximum has been taken for unit. The light-grades of A which, from 3.35 in the principal minimum, rise to the value 12.35 at a maximum, have been reduced to the interval of 3.00 to 12.00 for the sake of convenience.

t	I_B	I_{C_1}	I_{C_2}	σ_B	σ_{C_1}	σ_{C_2}
-30^h	0.7296	0.7525	0.7586	9.27	9.61	9.67
−24	.5836	.6019	.6674	7.40	7.73	8.60
−18	.4336	.4993	.5627	4.83	6.15	7.16
−12	.3661	.4275	.4506	3.55	4.85	5.29
− 6	.3484	.3487	.3500	3.10	3.13	3.16
0	.3433	.3433	.3433	3.00	3.00	3.00
+ 6	.3499	3488	.3477	3.15	3.12	3.11
+12	.3988	.4275	.4462	4.30	4.85	5.21
+18	.5306	.5591	.5586	6.67	7.11	7.10
+24	.6572	.6624	.6635	8.46	8.54	8.55
+30	.7644	.7528	.7553	9.70	9.61	9.64

In what follows we have tried, first of all, to give correct formulae for the derivation of a slightly excentric orbit from the variation of the light. These have then been used for the curve of ARGELANDER and for that of Dr. PANNEKOEK. Afterwards the spectroscopic data of BELOPOLSKY have also been freshly reduced, because there is some uncertainty about the resulting orbit [1]). This is perhaps to be attributed to the method of LEHMAN—FILHÉS [2]). This method is excellent for a satisfactory determination of the excentricity if it is large; but it is less suitable for a very small excentricity. For the drawing of the graphical velocity-curve remains always slightly arbitrary and this fact exerts too strong an influence in the case that e is small.

3. We thus start from the following hypothesis:

[1]) In the "Recherches nouvelles" (Memorie etc.) BELOPOLSKY gives $e = 0.04$; in his "New investigations" (A. J.) $e = 0.07$, as the result of the same observations.

[2]) A. N. n. 3242.

Two similar, elongated revolution-ellipsoids move about their common centre of gravity in elliptic orbits. We assume that the major axes of the ellipsoids are continually in each other's prolongation while their centres move about their common centre of gravity in obedience to the laws of KEPLER. Required the intensity of the light as it appears to our eye, if we assume that the ellipsoids may be exchanged for their uniformly illuminated projections on the sphere.

As unit of length we take the semi major axis of the larger ellipsoid (E_1); as unit of brightness the maximum of β-Lyrae.

Further let be:

$\varkappa$ the semi major axis of the smaller body:

q the proportion of the major axis to the diameter of the equator;

f the proportion of the major axis of the ellipse, which is the projection of one of the ellipsoids on the sphere, to the major axis of that same ellipsoid;

a the semi major axis of the relative orbit of the smaller body (E_2), e the excentricity, v the true anomaly, r the radius vector in the true relative orbit.

β the angle formed by the radius vector in the true orbit with the projection of the visual line on the plane of the orbit (on the further side of the sphere); this angle increases with the motion in the orbit;

ω the longitude of the periastron, counted in the same way as β;

i the angle between the plane of the orbit and a plane tangent to the sphere;

ϱ the projection of r on the sphere;

M the common part of two circles the radii of which are resp. $= 1$ and $= \varkappa$, having their centres at a distance of $\varrho' = \frac{\varrho}{f}$;

λ the proportion of the brightness (per unit of surface) of the larger of the elliptic projections to the smaller one;

J the apparent total light-intensity at the time t, (as seen from the earth).

As long as E_1 and E_2 do not cover each other, we have:

$$J = f.$$

When E_2 is covered by E_1

$$J = f\left(1 - \frac{M}{\pi(\lambda + \varkappa^2)}\right).$$

When E_1 is covered by E_2

$$J = f\left(1 - \frac{\lambda M}{\pi(\lambda + \varkappa^2)}\right).$$

Let $2\varphi'$ and 2φ be the angles formed by the common chord of the circles, which define M, as seen from their respective centres, then

$$M = \tfrac{1}{2}\{(2\varphi' - \sin 2\varphi') + \varkappa^2 (2\varphi - \sin 2\varphi)\}$$

$$\sin \varphi' = \varkappa \sin \varphi; \qquad \cos \varphi = \frac{\varrho'^2 + \varkappa^2 - 1}{2\varkappa \varrho'}.$$

φ' is always $< \frac{\pi}{2}$; φ may become $= \pi$, in the case that the smaller disc is seen projected wholly within the larger one.

Furthermore:

$$\varrho^2 = r^2 (1 - \cos^2 \beta \sin^2 i); \qquad \beta = \omega + v.$$

These formulae agree with those of Dr. Myers.

Computation of f.

The equation of the cylindre, enveloping the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ the axis of which makes the angles φ, χ, ψ with the X-, Y- and Z-axis, is:

$$\left(\frac{\cos^2 \varphi}{a^2} + \frac{\cos^2 \chi}{b^2} + \frac{\cos^2 \psi}{c^2}\right)\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1\right) =$$

$$= \left(\frac{x \cos \varphi}{a^2} + \frac{y \cos \chi}{b^2} + \frac{z \cos \psi}{c^2}\right)^2.$$

The surface of an orthogonal section of this cylindre is:

$$\Omega = \pi \sqrt{a^2 b^2 \cos^2 \psi + b^2 c^2 \cos^2 \varphi + c^2 a^2 \cos^2 \chi}.$$

or, putting

$$a = b = \frac{1}{q}, \; c = 1$$

$$\Omega = \frac{\pi}{q} \sqrt{1 - \varepsilon^2 \cos^2 \psi}, \qquad \varepsilon^2 = \frac{q^2 - 1}{q^2}.$$

The semi minor axis of the section is $\frac{1}{q}$, therefore the semi major axis

$$f = \sqrt{1 - \varepsilon^2 \cos^2 \psi}; \qquad \cos^2 \psi = \cos^2 \beta \sin^2 i;$$

because ψ is the angle between the major axis of the ellipsoid and the visual line.

In the computation of f Dr. Myers, instead of taking the instantaneous projection of the ellipsoid on the sphere, takes the intersection of the ellipsoid with a plane through the centre, at right angles to the visual line. In his opinion this is allowable "wenn die Abplattung

nicht ungeheuer gross ist". He therefore puts

$$f = \frac{1}{\sqrt{1 + (q^2 - 1)\cos^2 \psi}}.$$

As the rigorous formula is at least equally simple there is no reason for the substitution. If we put the expression of MYERS $= f'$, we have

$$\frac{f}{f'} = \sqrt{1 + \left(\frac{q^2-1}{2q}\right)^2 \sin^2 2\psi}.$$

Greatest values, for $\psi = \frac{\pi}{4}$

q	1.2	1.3	1.4	1.5
$\frac{f}{f'}$	1.02	1.03	1·06	1.08

There thus is introduced a systematic error, which, already for small elongations, cannot be neglected.

4. As soon as, with the aid of provisional elements, a light-curve has been calculated, we try to vary these elements in such a way that the differences between observation and computation are diminished. We have to investigate, therefore, in what way the light-intensity varies with the elements.

We have already: $J = F(f, M, \lambda, \varkappa^2)$. We will now, first of all, express df and dM in function of $d(\varkappa^2), d\beta, d\varrho$.

In the first place we have to consider that λ is fully determined by $\varkappa^2$, in the case, which as we shall presently see must be admitted, that during the minima E_2 is projected wholly on E_1. For, if $f_m = \sqrt{1-\varepsilon^2 \sin^2 i}$ (= value of f in both the minima), then, on the same supposition:

$$f_m \cdot \frac{\lambda}{\lambda + \varkappa^2} = \text{const.}_1 \text{ (= intensity at the principal minimum)}$$

$$f_m \left(1 - \frac{\varkappa^2 \lambda}{\lambda + \varkappa^2}\right) = \text{const.}_2 \text{ (= } \text{,,} \quad \text{,,} \quad \text{,,} \text{ secondary} \quad \text{,,} \text{)}$$

From these, after division:

$$d\lambda = \frac{\lambda(1-\lambda)}{\varkappa^2} d(\varkappa^2) \text{ and } \frac{df_m}{f_m} = \frac{\lambda}{\lambda + \varkappa^2} d\varkappa^2 = -\frac{d(\varepsilon^2 \sin^2 i)}{2{f_m}^2}.$$

With the aid of the latter formula we get without difficulty

$$df = \frac{\varepsilon^2 \sin^2 i}{2f} \sin 2\beta d\beta + \frac{\cos^2 \beta}{f}(1 - \varepsilon^2 \sin^2 i) \frac{\lambda}{\lambda + \varkappa^2} d(\varkappa^2);$$

which is independent of the variation of i.

The computation of dM directly from the formulae is rather lengthy [1]); by considering the geometrical meaning of M, dM is found at once. Evidently M is purely a function of $\varkappa$ and ϱ'. If $\varkappa$ increases by the amount $\Delta\varkappa$, the increment of M is a strip $2\varkappa\varphi\Delta\varkappa$; if ϱ' increases by $\Delta\varrho'$, the increment of M is *negative* and equal to a strip (crescent) $2 \sin\varphi' . \Delta\varrho' = 2\varkappa \sin\varphi . \Delta\varrho'$. Therefore

$$dM = \varphi d(\varkappa^2) - 2 \sin\varphi' . d\varrho'$$

$$= \varphi d(\varkappa^2) - \frac{2 \sin\varphi'}{f} d\varrho + \frac{2\varrho \sin\varphi'}{f^2} . df$$

If in this expression we substitute the value, given above, of df, we get dM expressed as a function of $d(\varkappa^2)$, $d\beta$, $d\varrho$.

5. *Calculation of* $d\beta$ *and* $d\omega$ *in function of the variations of the elements of the orbit and of the epoch.*

If $\sin\varphi = e$, then (*vide* BAUSCHINGER, die Bahnbestimmung der Himmelskörper, n°. 197):

$$dv = \left(\frac{a}{r}\right)^2 \cos\varphi \{(t-T)\, d\mu - \mu dT\} + \frac{a}{r} \cos\varphi \sin v \left(1 + \frac{r}{p}\right) d\varphi.$$

$$\frac{d\varrho}{\varrho} = \frac{da}{a} - \sin(P-\Omega) \cos(P-\Omega) \sin i \operatorname{tg} i . d\omega - \sin^2(P-\Omega) \operatorname{tg} i . di$$

$$+ \left(\frac{a}{r}\right)^2 \{e \sin E - \sin(P-\Omega) \cos(P-\Omega) \sin i \operatorname{tg} i \cos\varphi \{(t-T)\, d\mu - \mu dT\}$$

$$- \left(\frac{a}{r}\right)^2 \left\{(\cos E - e) \cos\varphi + \sin(P-\Omega) \cos(P-\Omega) \sin i \operatorname{tg} i \sin E \left(\frac{r}{a} + \cos^2\varphi\right)\right\} d\varphi.$$

According to the definition of ω adopted above, we have to put:

$$\varrho \sin(P-\Omega) = r \cos i \cos(\omega + v) = r \cos i \cos\beta$$

$$\varrho \cos(P-\Omega) = -r \sin(\omega + v) = -r \sin\beta$$

We now pass to the following particular case:

a. the original orbit is circular;

b. if $i = 90° - \iota'$, then ι' is so small that 3rd and higher powers may be neglected; the same is true for $\sin\varphi$.

Furthermore let $d\mu = 0$; $d\omega = 0$; $2e \cos\omega = x$, $2e \sin\omega = y$; $n = \frac{2\pi}{U}$ (U = period = 12.91 days), t_1 = time counted from "superior conjunction"; $dM_0 = -\mu dT$.

Then with sufficient approximation

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[1]) See: Untersuchungen über den Lichtwechsel des Sternes β Persei, von J. HARTING. (München 1889) p. 41.

$$d\beta = dM_0 + x \sin nt_1 - y \cos nt_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$\frac{d\varrho}{\varrho} = \frac{da}{a} + \frac{a^2}{2\varrho^2} \cos^2 nt_1 \, i'^2 + \frac{a^2}{2\varrho^2} \sin 2nt_1 \, . \, dM_0 -$$

$$- \tfrac{1}{2} x \left(\cos nt_1 - \frac{a^2}{\varrho^2} \sin 2nt_1 \sin nt_1 \right) - \tfrac{1}{2} y \left(\sin nt_1 + \frac{a^2}{\varrho^2} \sin 2\, nt_1 \sin nt_1 \right) . \quad (b)$$

As these differential expressions have led several astronomers [1]) into error, we will derive them in still another way.

From:

$$\beta = v + \omega$$

we get:

$$d\beta = dv + d\omega.$$

In the circular orbit $v = M$; in the elliptic orbit this becomes:

$$v = M + 2e \sin M + \ldots + dM_0.$$

If we substitute $M = nt_1 - \omega$, and put $d\omega = 0$, we get, neglecting higher powers of e:

$$d\beta = dM_0 + x \sin nt_1 - y \cos nt_1$$

If in: $\varrho^2 = r^2 \sin^2 \beta + r^2 \cos^2 i \cos^2 \beta,$

$$i = 90° - i',$$

then, neglecting higher powers of i':

$$\frac{d\varrho}{\varrho} = \frac{dr}{r} + \frac{r^2}{2\varrho^2} \sin 2\beta \, d\beta + \frac{r^2}{2\varrho^2} \cos^2 \beta . \, i'^2 \quad . \quad . \quad . \quad . \quad (c)$$

In the elliptic orbit we have:

$$r = \frac{a(1-e^2)}{1 + e \cos v} = a_0 + da - ae \cos (\beta - \omega) + \ldots =$$

$$= a_0 + da - \frac{1}{2} a x \cos nt_1 - \frac{1}{2} ay \sin nt_1 \ldots$$

Therefore:

$$dr = da - \frac{1}{2} ax \cos nt_1 - \frac{1}{2} ay \sin nt_1$$

and, substituting this in (c), we get the expression already given of $\frac{d\varrho}{\varrho}$.

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[1]) Dr. Myers puts $d\beta = 0$ for $t_1 = 0$ and at the same time $dM_0 = 0$; this is incompatible with (a). Prof. Hartwig, in his paper: "Der veränderliche Stern vom Algoltypus Z Herculis" (Bamberg 1900) p. 39, puts $\sin (P\text{-}\mathfrak{h}) \cos (P\text{-}\mathfrak{h}) \sin i \, tg \, i = 0$ for $i = 90°$, whereas, according to our formulas, it becomes $- \frac{r^2}{2\varrho^2} \sin 2\beta$. (See also A.N. 3644).

Dr. Pannekoek quotes another instance in his Thesis on Algol (p. 22—3).

6. We thus have consecutively expressed dM and df in function of $d(\varkappa^2)$, $d\beta$ and $d\varrho$; and afterwards $d\beta$ and $d\varrho$ in function of da, i'^2, dM_0, x and y. If now we differentiate the expression

$$J_1 = f\left(1 - \frac{M}{\pi(\lambda + \varkappa^2)}\right),$$

valid in the vicinity of the first minimum, we find, by consecutive substitution, the following expression for dJ_1:

$$\pi(\lambda + \varkappa^2)\, dJ_1 = K_1\, d(\varkappa^2) + A_1\, da + I_1\, i'^2 + X_1 x + Y_1 y + \Delta_1 (dM_0 - y),$$

in which:

$$K_1 = \frac{\pi\lambda}{q^2 f^2} J_1 \cos^2 nt_1 + \pi\left(1 + \frac{\lambda - \lambda^2}{\varkappa^2}\right)(f - J_1) - f\varphi - \frac{2\lambda}{q(\lambda+\varkappa^2)} \cdot \frac{\varrho}{f^2} \cdot \cos^2 nt_1 \sin\varphi' ;$$

$$A_1 = \frac{2\varrho \sin\varphi'}{r} \; ; \; I_1 = \frac{r^2 \cos^2 nt_1}{\varrho} \sin\varphi' ;$$

$$X_1 = \Delta_1 \sin nt_1 - \varrho \sin\varphi' \cos nt_1 \; ; \; Y_1 = \Delta_1 (1 - \cos nt_1) - \varrho \sin\varphi' \sin nt_1 ;$$

$$\Delta_1 = \frac{r^2 \sin 2nt_1 \sin\varphi'}{\varrho} + \frac{\varepsilon^2}{2f^2} \sin 2nt_1 \{\pi(\lambda + \varkappa^2) J_1 - 2\varrho \sin\varphi'\} .$$

If we treat in the same way the expression:

$$J_2 = f\left(1 - \frac{\lambda M}{\pi(\lambda+\varkappa^2)}\right),$$

valid in the vicinity of the second minimum, we find, putting $t_2 = t_1 - \frac{U}{2}$:

$$\pi \frac{\lambda+\varkappa^2}{\lambda}\, dJ_2 = K_2\, d(\varkappa^2) + A_2\, da + I_2 i'^2 + X_2 x + Y_2 y + \Delta_2 (dM_0 + y)$$

in which:

$$K_2 = \frac{\pi}{q^2 f^2} J_2 \cos^2 nt_2 + \pi(f - J_2) - f\varphi - \frac{2\lambda}{q(\lambda + \varkappa^2)} \cdot \frac{\varrho}{f^2} \cdot \cos^2 nt_2 \sin\varphi' ;$$

$$A_2 = \frac{2\varrho \sin\varphi'}{r} \; ; \; I_2 = \frac{r^2 \cos^2 nt_2}{\varrho} \sin\varphi' ;$$

$$X_2 = -\Delta_2 \sin nt_2 + \varrho \sin\varphi' \cos nt_2 ; \; Y_2 = -\Delta_2 (1 - \cos nt_2) + \varrho \sin\varphi' \sin nt_2$$

$$\Delta_2 = \frac{r^2 \sin 2nt_2 \sin\varphi'}{\varrho} + \frac{\varepsilon^2}{2f^2} \sin 2nt_2 \left(\pi \frac{\lambda + \varkappa^2}{\lambda} J_2 - 2\varrho \sin\varphi'\right).$$

7. If the observations do not give the light-intensity, but the brightness expressed in magnitudes or in grades, then we have still to express the variation of the number indicating the magnitude or the grade, in the variation of the light-intensity.

Let J_0 represent the intensity at the maximum, G_0 the corresponding magnitude, J and G the same quantities at the time t, then, by the formula of POGSON:

$$G - G_0 = 2.512\ (log\ J_0 - log\ J).$$

Consequently:

$$dG = -2.512\ m.\frac{dJ}{J},\ (m = modulus\ of\ Brigg's\ log.)$$

$$dG = -1.092\ \frac{dJ}{J}.$$

Now, if $-\frac{1}{\nu}$ is the equivalent in magnitudes of a grade, then, σ_0 and σ being the number of grades:

$$\sigma_0 - \sigma = \nu(G - G_0) = 2.512\nu\ (log\ J_0 - log\ J)$$

Therefore:

$$d\sigma = 1.092\nu\frac{dJ}{J}$$

Putting the value of ARGELANDER's grade for the light-curve of β Lyrae at 0.130 magnitudes, then:

$$d\sigma = 8.413\ \frac{dJ}{J}.$$

8. In the hypothesis which we adopted, the main phases (min.$_1$, max.$_1$, min.$_2$, max.$_2$) take place for the values $\beta = 0$, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ of β_1. Let v_1, v_2, v_3, v_4 represent the true anomalies for these values; M_1, M_2, M_3, M_4 the corresponding mean anomalies. If, as is the case with β Lyrae, the intervals are nearly equal, e must be small and we may put approximately:

$v_1 = M_1 - y$; $v_2 = M_2 + x$; $v_3 = M_3 + y$; $v_4 = M_4 - x$.

$(x = 2e\ cos\ \omega$; $y = 2e\ sin\ \omega)$

or:

$$v_2 - v_1 = \frac{\pi}{2} = (M_2 - M_1) + x + y$$

$$v_3 - v_2 = \frac{\pi}{2} = M_3 - M_2) - x + y$$

$$v_4 - v_3 = \frac{\pi}{2} = (M_4 - M_3) - x - y$$

If the differences $M_2 - M_1 \ldots$ [1]) are known with equal and

[1]) The time-equation for the reduction to the common centre of gravity, computed from the spectroscopic orbit, is found to reach a value of somewhat over $\pm$ 100 seconds and may consequently be neglected.

sufficient precision, we find from these formulae the most probable values of x and y as follows:

$$\left.\begin{aligned} 4x &= -\pi + (M_4 - M_3) + 2(M_3 - M_2) - (M_2 - M_1) \\ 4y &= \quad \pi + (M_4 - M_3) - 2(M_3 - M_2) - (M_2 - M_1) \end{aligned}\right\} \quad I$$

If we combine only similar phases, we get

$$\left.\begin{aligned} 2x &= -\pi + (M_4 - M_3) \\ 2y &= \quad \pi - (M_3 - M_1) \end{aligned}\right\} \quad II$$

The two solutions are identical, if

$$(M_2 - M_1) + (M_4 - M_3) = \pi.$$

In A. N. n°. 3456 Dr. PANNEKOEK summarises the intervals, counted from the principal minimum, for different observers between 1842 to 1895.

Dividing this period in two, he finds on an average: ($U = 12^d.91$)

	max_1—min_1	min_2—min_1	max_2—min_1
1842—1870	$3^d.12$	$6^d.40$	$9^d.54$
1870—1895	$3^d.32$	$6^d.48$	$9^d.73$.

From these values we find, for the first period:

according to form. (I): according to form. (II):

$$\left.\begin{aligned} e\sin\omega &= -0\ 0052; \quad e = 0.009 \\ e\cos\omega &= +0.0076; \quad \omega = 326° \end{aligned}\right. \qquad \left.\begin{aligned} e\sin\omega &= +0.0067; \quad e = 0.008 \\ e\cos\omega &= -0.0043; \quad \omega = 123°. \end{aligned}\right.$$

Similarly for the second period:

$$\left.\begin{aligned} e\sin\omega &= +0.0040; \quad e = 0.013 \\ e\cos\omega &= -0.0125; \quad \omega = 162° \end{aligned}\right. \qquad \left.\begin{aligned} e\sin\omega &= -0.0030; \quad e = 0.006 \\ e\cos\omega &= -0.0055; \quad \omega = 209° \end{aligned}\right.$$

The only conclusion to be derived from these results is that e was very minute in both periods, and hardly exceeding 0.01.

9. A single glance at the numbers communicated by Dr. PANNEKOEK shows that a trial to derive something more definite from the results of the *separate* observers would be quite hopeless. In particular we may allege the considerable difference between the results obtained by LINDEMANN and PANNEKOEK, in their reduction of the observations of PLASSMANN. It thus seems to be out of place, from these observations alone, to draw the conclusion that the excentricity has increased.

Dr. L. TERKAN has proposed the following method of deriving the inclination of the orbit.[1])

[1]) A. N. nr 4067.

The minimum or maximum of light takes place when ϱ takes "extreme" values, consequently when

$$\frac{d\varrho}{dv} = \cos v \sin v \sin^2 i + e \sin v = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In formula (2) $\sin v = 0$ for the principal minimum, $\cos v = -\frac{e}{\sin^2 i}$ for the "secondary maximum". This is in the assumption that the time of the principal minimum coincides with the time of periastron. Therefore if, at the moment of the maximum, we know e and v, then we know also i. TERKAN adopts the value 0.07, derived for e by BELOPOLSKY from his spectroscopic observations [1]). He determines the mean anomaly at the maximum from the interval found by PLASSMANN [2]):

$$\text{II min.} - \text{II max.} = 3{,}05 \text{ days},$$

and then expands this anomaly in a series [3]) by the aid of $\cos v = -\frac{e}{\sin^2 i}$. This series has an argument α, which contains $\sin^2 i$. He thus finds

$$i = 51°.3.$$

Afterwards, in his Hungarian paper [4]), he takes $e = 0.06$. From his own observations he derives: I min. — I max. = 3,48 days and then finds, using the usual equations of KEPLER, by the aid of $\cos v = -\frac{e}{\sin^2 i}$:

$$i = 30°.$$

Even if we disregard the very doubtful value of the numerical data, the hypothesis seems unfounded that the maximum of the light occurs at the moment that ϱ is a maximum. If, moreover, we assume with Dr. TERKAN, both the celestial bodies to be spherical, then the light must be constant as long as the two spheres do not cover each other as seen by the observer. This is not confirmed by observation. Besides there can be no question of a clearly defined epoch of maximum in such a case. The way in which Dr. TERKAN meets this objection by saying: "that our eye or the telescope is unable to separate the system and that the rays of light which in space come

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[1]) See p. 462. 1st footnote.

[2]) A. N. nr 3242.

[3]) In this series e has been erroneously substituted for $\sin\varphi\cos\varphi$ and $\frac{\sqrt{4-e^2}-e}{\sqrt{4-e^2}+e}$ (instead of $\sqrt{\frac{1-e}{1+e}}$) for $tg\,(45°-\frac{1}{2}\varphi)$.

[4]) β Lyrae palyaelemeinek etc. p. 412.

from the same distance, but from a larger field, are united to a larger disc" [1]), seems little satisfactory.

10. *Determination of the elements of the orbit etc. by means of the light-curve of* ARGELANDER [2]).

As a first approximation we put $i = 90°$, $e = 0$.

An approximate value of q is furnished by the general course of the curve in the vicinity of the maxima. As long as it is symmetrical in regard to the ordinate of the maximum, we may assume that the eclipse has not yet begun, so that

$$J = \sqrt{(1-\varepsilon^2 \sin^2 nt_M)};$$

t_M being the time counted from the maximum.

From the light-curve we take the decrease of $(\sigma_0-\sigma)$ in grades, for equal intervals of time before and after the two maxima.

t_M	$(\sigma_0-\sigma)_I$	$(\sigma_0-\sigma)_{II}$
-30^h	0.76	0.50
−24	0.47	0.31
−18	0.25	0.18
−12	0.10	0.07
− 6	0.02	0.02
+ 6	0.03	0.03
+12	0.09	0.11
+18	0.21	0.24
+24	0.38	0.43
+30	0.59	0.67

t_M	$(\sigma_0-\sigma)_{mean}$
$\pm 6^h$	0.025
±12	0.093
±18	0.220

t_M	$(O-C)_I$	$(O-C)_{II}$
-18^h	−0.03	+0.04
−12	0.00	+0.03
− 6	0.00	0.00
+ 6	−0.01	−0.01
+12	+0.01	−0.01
+18	+0.01	−0.01

An increasing dissymmetry begins to show itself for both the maxima at about 24 hours distance from these epochs.

With the mean values $(\sigma_0-\sigma)_{mean}$ the light-intensities were now computed by the formula:

$$\sigma_0-\sigma = -\frac{2.512}{0.13} \log J$$

and the relation

$$\varepsilon^2 \sin^2 nt_M = 1-J^2$$

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[1]) β Lyrae palyaelemeinek etc. p. 417.

[2]) De Stella β Lyrae variabili commentatio altera. Scripsit FREDERICUS ARGELANDER Bonnae a. 1859. — Curva "vera" pro 1850.

furnished the data:

$$0.015\ \varepsilon^2 = 0.006$$
$$0.058\ \varepsilon^2 = 0.022$$
$$0.127\ \varepsilon^2 = 0.051$$

leading to the most probable values $\varepsilon^2 = 0.397$; $q = 1.288$.

The deviations Obs.-Comp. have been given in the two last columns.

Having found q, we get $\varkappa$ and λ from the light-intensities at the two minima. The values of these being 0.3433 and 0.6365, we obtain, for $i = 90°$, the two relations

$$\frac{1}{q} \cdot \frac{\lambda}{\lambda + \varkappa^2} = 0.3433; \quad \frac{1}{q}\left(1 - \frac{\varkappa^2\lambda}{\lambda + \varkappa^2}\right) = 0.6365,$$

whence:

$$\varkappa = 0.6387; \quad \lambda = 0.3233.$$

Finally, at the moment at which the eclipse begins:

$$\frac{\varrho}{f} = \varrho' = 1 + \varkappa.$$

The consideration of the asymmetry, shows that this must be the case shortly after 18^h ($ntM = \pm 20°55'$). We therefore put:

$$\frac{\varrho}{f} = \frac{a \cos 21°}{\sqrt{1 - \varepsilon^2 \sin^2 21°}} = 1.6387 = 1 + \varkappa$$

from which:

$$a = 1.710.$$

We thus have, as a first approximation, the following elements: $\varkappa = 0.6387$; $\lambda = 0.3233$; $q = 1.288$; $a = 1.710$; $e = 0$; $i = 90°$ and, as for the "epoch", we assume, that the central eclipse of E_2 by E_1, coincides with the principal minimum of ARGELANDER's curve.

11. In the following table the 2nd column, headed O_1, shows the light-grades of ARGELANDER's curve for equal intervals before and after the principal minimum; the 7th column, headed O_2, similarly shows the same element before and after the half period $= 6^d.455$ (*not* therefore before and after the secondary minimum, which ARGELANDER places at $6^d.375$ from the principal minimum). The columns C_{s_1} and C_{s_2} contain the light-grades, computed by the aid of the elements given just now.

12. As will be remarked, the deviations $O—C_s$ are in the main negative before, positive after the two minima. We conclude that, by shifting the theoretical light-curve in a negative direction with regard to the time, we may obtain improved agreement. The excen-

t	O_1	C_{S_1}	C_{M_1}	$O_1-C_{S_1}$	$O_1-C_{M_1}$	O_2	C_{S_2}	C_{M_2}	$O_2-C_{S_2}$	$O_2-C_{M_2}$
—72	11.95	11.98	11.98	—0.03	—0.03	11.87	11.98	11.98	—0.11	—0.11
—66	11.84	11.91	11.93	—0.07	—0.09	11 79	11.91	11.93	—0.12	—0.14
—60	11.69	11.80	11.84	—0.11	—0.15	11.66	11.80	11.84	—0.14	—0.16
—54	11.48	11.58	11.72	—0.10	—0.24	11.48	11.62	11.72	—0.14	—0.24
—48	11.20	11.34	11.45	—0.14	—0.25	11 25	11.37	11.52	—0.12	—0.27
—42	10.82	10.75	11.04	+0 07	—0.22	10.99	11.06	11.26	—0.07	—0.27
—36	10.29	10.07	10.44	+0.22	—0.15	10.68	10.68	10.92	0.00	—0.24
—30	9.27	9.14	9.64	+0.13	—0.37	10.30	10.22	10.50	+0.08	—0.20
—24	7.40	7.91	8.55	—0.51	—1.15	9 78	9.70	10 00	+0.08	—0.22
—18	4.83	6.28	7.13	—1.45	—2.30	9.04	9.13	9.44	—0.09	—0.40
—12	3.55	4.22	5.26	—0.67	—1.71	8.42	8.56	8.82	—0.14	—0.40
— 6	3.10	3.05	3.15	+0.05	—0.05	8.21	8.23	8.25	—0.02	—0.04
0	3.00	3.00	3.00	0.00	0.00	8.20	8.19	8.19	+0.01	0.00
+ 6	3.15	3.05	3.15	+0.10	0.00	8 38	8.23	8 25	+0.15	+0.13
+12	4 30	4.22	5.26	+0.08	—0.96	8.91	8.56	8.82	+0.35	+0.09
+18	6.67	6.28	7.13	+0.39	—0.46	9.64	9.13	9.44	+0.51	+0.20
+24	8.46	7.91	8.55	+0.55	—0 09	10.25	9.70	10.00	+0.55	+0.25
+30	9.70	9.14	9.64	+0 56	+0.06	10.75	10.22	10.50	+0.53	+0.25
+36	10.50	10.07	10 44	+0.43	+0.06	11.14	10.68	10.92	+0.46	+0.22
+42	10.97	10.75	11.04	+0.20	—0.07	11.43	11.06	11.26	+0.37	+0.17
+48	11.31	11.34	11.45	—0.03	—0.14	11.64	11.37	11.52	+0.27	+0.12
+54	11.57	11.58	11.72	—0.01	—0.15	11.79	11.62	11.72	+0.17	+0.07
+60	11.75	11.80	11.84	—0.05	—0.09	11.91	11.80	11.84	+0.11	+0.07
+66	11.88	11.91	11.93	—0.03	—0.05	11.98	11.91	11.93	+0.07	+0.05
+72	11.91	11.98	11.98	—0.07	—0.07	12.01	11.98	11.98	+0.03	+0.03

tricity, besides displacing the maxima and minima, also causes a slight dissymmetry in regard to the minima. In order to separate the influence of the excentricity on the asymmetry from that of the epoch, we may divide the equations of condition into two groups. For the coefficients K, A, I and X are even functions; Y and Δ uneven functions of nt_1, resp. nt_2. If, therefore, we take the sum and the difference of two equations of condition, corresponding to

the times $+ t_1$ and $- t_1$, the former of the resulting equations will only contain the quantities $d(x^2)$, da, ι'^2 and x, the latter only y and dM.

In this way the following equations have been derived for successive intervals of six hours counted from the 1th, resp. 2nd minimum.

They do not rest however on the above elements but on those

PRINCIPAL MINIMUM.

t_1

$\pm 72^h$	$+ 0.03\ d(x^2)$	$+ 0.00\ da$	$0.00\ (x\iota'^2)$	$+ 0.70\ x$	$= - 0.05$	$0.62\ y$	$+ 0.70\ (dM_0 - y)$	$= - 0.02$
66	+ .13	.00	.00	1.41	.07	1.12	1.45	.02
60	.30	.00	.00	2.02	.12	1.41	2.15	.03
54	.04	.60	.02	2.59	.19	1.55	3.20	.04
48	.40	1.46	.06	3.16	.19	0.94	4.76	.05
42	.50	1.92	.14	3.55	.14	.76	6.37	.07
36	.40	2.34	.28	3.80	.04	.60	8.18	.10
30	.06	2.58	.50	3.89	.15	.44	10.32	.21
24	.60	2.70	.91	3.78	.62	.31	12.92	.53
18	1.75	2.60	1.70	3.43	1.38	.18	16.08	.92
12	3.42	2.20	3.38	2.69	1.33	.07	19.57	.37
6	2.97	0.79	4.98	1.02	0.09	.01	14.11	.02

SECONDARY MINIMUM.

t_2

$\pm 72^h$	$+ 0.03\ d(x^2)$	$+ 0.00\ da$	$0.00\ (x\iota'^2)$	$- 0\ 05\ x$	$= - 0.04$	$- 0.04\ y$	$+ 0.05\ (dM_0 + y)$	$= .02$
66	.12	.00	.00	.10	.04	.07	.10	.09
60	.30	.00	.00	.14	.05	.09	.15	.12
54	.33	.24	.00	.26	.08	.01	.40	.15
48	.30	.58	.02	.44	.07	.07	.90	.19
42	.38	.78	.05	.59	.05	.06	1.42	.22
36	.52	.89	.10	.71	.01	.05	2.01	.23
30	.70	.94	.18	.79	.02	.04	2.65	.22
24	.90	.91	.30	.80	.01	.03	3.35	.23
18	1.11	.80	.52	.73	.10	.01	4.02	.30
12	1.22	.58	.89	.55	.15	.00	4.48	.24
6	0.62	.17	1.09	.17	.04	.00	2.71	.08

which have been derived by repeated approximations from ARGELANDER's curve by Dr. MYERS. In his opinion these are the best possible circular elements:

$$a = 1\ 8955\ ;\ \varkappa = 0.7580\ ;\ q = 1.1993\ ;\ \lambda = 0.4023\ ;\ i = 0$$

By their aid I computed the light-grades C_{M_1} and C_{M_2} of the preceding table. As will be remarked, the deviations $O - C_{M_1}$ are rather considerable in the vicinity of the principal minimum.

In deriving the following normal equations, the equations of condition for $t_1 = \pm 6^h$ and $t_2 = \pm 6^h$ have been neglected.

Normal-equations:

$$\begin{aligned}
&18.54\, d(\varkappa^2) + 15.17\, da + 16.98\, (10\iota'^2) + 15.31\, x = -\ 7.670\\
&15.17\ ,, + 41.68\ ,, + 18.14\ ,, + 53.17\ ,, = -\ 9.673\\
&16.98\ ,, + 18.14\ ,, + 20.29\ ,, + 20.75\ ,, = -\ 7.720\\
&15.31\ ,, + 53.17\ ,, + 20.75\ ,, + 101.96\ ,, = -\ 13.227\\
&255.69\, y + 160.37\, (dM_0 - y) = 10.242\\
&160.37\, y + 1125.52\, (dM_0 - y) = 37.759
\end{aligned}$$

Solution of the first four equations:

$$x = -\ 0.026\ ;\ 10\iota'^2 = -\ 0.044\ ;\ da = -\ 0.0741\ ;\ d(\varkappa^2) = -\ 0.2915.$$

As ι' becomes imaginary, we put $\iota' = 0$ in the equations of condition, and then find:

$$x = -\ 0.026\ ;\ da = -\ 0.0799\ ;\ d(\varkappa^2) = -\ 0.3268.$$
$$y = +\ 0{,}021\ ;\ dM_0 - y = +\ 0.031,$$

which lead to the improved elements:

$$a = 1.8156\ ;\ \varkappa = 0.4978\ ;\ \lambda = 0.2249\ ;\ q = 1\ 3859\ ;\ e = 0.017;\ \omega = 141°.3.$$

The correction for $\varkappa^2$ is particularly large, more than half its original value (0.5746). As in such a case dJ cannot any longer be considered to be proportinal to $d(\varkappa^2)$, we should have to compute a new light-curve by the aid of the new elements; we should then have to calculate the differential coefficients in order — if necessary — to find a new approximation.

In the following table the columns C_1 and C_2 show the light-grades calculated by means of the improved elements.

In fig. I has been given a graphical representation of these numbers. The agreement in the vicinity of the principal minimum is considerably improved. It is true that there remains a deviation exceeding a light-grade, at 18 hours before the minimum. It might perhaps be further diminished by a repetition of the whole process. If, however, we take into account the uncertainty mostly existing when we draw the curve for the vicinity of the minimum, then it seems hardly worth while to repeat the elaborate calculation. At all

t	C_1	O_1	O_1-C_1	O_2	C_2	O_2-C_2
—72h	11.95	11.95	0.00	11.87	11.97	—0.10
—66	11.84	11.84	.00	11.79	11.88	— .09
—60	11.69	11.68	+ .01	11.66	11.73	— .07
—54	11.48	11.47	+ .01	11.48	11.52	— .04
—48	11.20	11.22	— .02	11.25	11.26	— .01
—42	10.82	10.93	— .11	10.99	10.96	+ .03
—36	10.29	10.42	— .13	10.68	10.55	+ .13
—30	9.27	9.47	— .20	10.30	10.05	+ .25
—24	7.40	8.06	— .66	9.78	9.47	+ .31
—18	4.83	6.01	—1.18	9.04	8.84	+ .20
—12	3.55	3.46	+ .09	8.42	8.35	+ .07
— 6	3.10	3.05	+ .05	8.21	8.22	— .01
0	3.00	3.00	.00	8.20	8 21	— .01
+ 6	3.15	3.06	+ .09	8.38	8.30	+ .05
+12	4.30	3.88	+ .42	8.91	8.73	+ .18
+18	6.67	6.39	+ .28	9.64	9.35	+ .29
+24	8.46	8.32	+ .14	10.25	9.94	+ .31
+30	9.70	9.66	+ .04	10.75	10 45	+ .30
+36	10.50	10.54	— .04	11.14	10.87	+ .27
+42	10.97	11.01	— .04	11.43	11.18	+ .25
+48	11.31	11.30	+ .01	11.64	11.44	+ .20
+54	11.57	11.55	+ .02	11.79	11.66	+ .13
+60	11.75	11.75	.00	11.91	11.83	+ .08
+66	11.88	11.89	— .01	11.98	11.94	+ .04
+72	11.91	11.98	— .07	12.01	12.00	+ .01

events the small coefficients of y in the equations of condition, show clearly that it is impossible to explain any appreciable asymmetry by an excentricity of a few hundredths. The improved agreement near the principal minimum is obtained in the main by shifting the theoretical principal minimum by $\frac{dM_0-y}{2\pi}\times U = 0^d.063$ in the direction of the negative time-axis, while at the same time the secondary minimum occurs 0.069 days before the minimum of ARGELANDER's curve.

13. A second set of elements was derived by making the plausible assumption, that the first minimum in ARGELANDER's curve occurs 0.08 days earlier. We thus get rid of the greater part of the dissymmetry. The second theoretical minimum is assumed to coincide with the observed minimum. The interval between the two minima thus becomes just half the period ($6^d.375 + 0^d.08 = 6^d.455$). As a consequence the orbit must be either circular or elliptic with

t	O_1	C_1	C'_1	O_1-C_1	$O_1-C'_1$	O_2	C_2	C'_2	O_2-C_2	$O_2-C'_2$
— 72^h	11.97	11.97	11.97	0.00	0.00	11.89	11.97	11.97	—.08	—.08
— 66	11.88	11.89	11.83	—.01	+.05	11.83	11.89	11.95	—.06	—.12
— 60	11.74	11.75	11.66	—.01	+.08	11.71	11.75	11.84	—.04	—.13
— 54	11.54	11.56	11.44	— 02	+.10	11.54	11.56	11.67	— 02	—.13
— 48	11.29	11.31	11.18	—.02	+ 11	11.33	11.31	11.44	+ .02	—.11
— 42	10.95	10.98	10.86	—.03	+.09	11.08	11.01	11.15	+.07	—.07
— 36	10.48	10.35	10.16	+.13	+.32	10.78	10.62	10.71	+.16	+.07
— 30	9.65	9.42	9 20	+.23	+ 45	10.42	10.26	10.40	+.16	+.02
— 24	8.06	8.09	7.85	—.03	+.21	9.95	9.63	9.75	+.32	+.20
— 18	5.48	6.29	6.05	—.81	—.57	9.28	9.06	9.15	+.22	+.13
— 12	3.80	4.07	3.90	—.27	—.10	8.56	8.52	8.57	+.04	—.01
— 6	3.18	3.05	3.05	+.13	+.13	8.25	8.25	8.26	.00	—.01
0	3.00	3.00	3.00	.00	.00	8.19	8.20	8.20	—.01	—.01
+ 6	3.07	3.05	3.05	+.02	+.02	8.30	8.25	8.26	+.05	+.04
+ 12	3.76	4 07	3.90	—.31	—.14	8.71	8 52	8.57	+ .19	+ .14
+ 18	6.01	6.29	6.05	—.28	—.04	9.40	9.06	9.15	+.34	+.25
+ 24	7.98	8 09	7.85	— 11	+.13	10.08	9.63	9.75	+.45	+.33
+ 30	9.37	9.42	9.20	—.05	+.17	10.61	10.26	10.40	+.35	+.21
+ 36	10 31	10.35	10.16	—.04	+.15	11.04	10.62	10 71	+.42	+.33
+ 42	10.84	10.98	10.86	—.14	—.02	11.35	11.01	11.15	+.34	+.20
+ 48	11.22	11.31	11.18	—.09	+.04	11.58	11.31	11 44	+ 27	+.14
+ 54	11.50	11.56	11.44	—.06	+.06	11.75	11.56	11.67	+.19	+.08
+ 60	11 71	11.75	11.66	—.04	+.05	11.87	11.75	11.84	+.12	+.03
+ 66	11.84	11.89	11.83	—.05	+.01	11.96	11.89	11.95	+.07	+.01
+ 72	11.91	11.97	11.97	—.06	—.06	12.00	11.97	11.97	+.03	+.03

the major axis at right angles to the line of the nodes. In this way we find:

$a=1.7209$; $\varkappa=0.5015$; $\lambda=0.2276$; $q=1.3944$; $i'=7°,25$; $e=0.04$; $\omega=180°$.

In the next tables the columns C_1 and C_2 show the light-grades computed by means of the first five elements, neglecting the excentricity. The columns C_1 and C_2 contain the same quantities taking into account the excentricity.

The mean deviation of the values in the columns O_1-C_1' and O_2-C_2' is $\pm$ 0.17 light-grades, whereas ARGELANDER assigns the value $\pm$ 0.16 to the mean error of the ordinates of his light-curve (prob. error 0.1095). It would be quite illusory therefore to endeavour to obtain an improved agreement. Against the elliptic orbit there is however the grave objection that it gives the first maximum 0.18 days after —, the second maximum 0.10 days before the corresponding maxima of ARGELANDER's light-curve. In the circular orbit the first maximum lies only 0.02 days, the second 0.06 days later, whereas the agreement is still very satisfactory.

14. Finally we communicate a set of circular elements obtained by a repeated approximation from the light-curve of Dr. PANNEKOEK:

$$a = 1.5378\,;\ \varkappa = 0.5378\,;\ \lambda = 0.2900\,;\ q = 1.4609.$$

In deriving them we assumed that a cannot fall short of $1+\varkappa$. We further assumed the theoretical principal minimum to coincide with the observed minimum.

In the following table t is the number of hours before and after the *theoretical* principal and secondary minimum; O_1 and O_2 are the light-grades at the same moments, as read off from Dr. P.'s light-curve; C_1 and C_2 the light-grades of the theoretical curve.

The results have been graphically represented in Fig. II. The remaining deviations are mainly positive before the first minimum; after that they are negative. At the secondary minimum the signs are reversed. The deviations might be rather considerably diminished if, with a small excentricity ($e \sin \omega = 0.016$), we place the principal minimum in Dr. PANNEKOEK's light-curve 0.063 days later, the secondary minimum 0.069 earlier. In this way, however, the interval in time min. I — min. II is diminished more considerably than seems admissible.

For the rest it need not be said, that in the present case, where two gaseous bodies seem to be in contact, the Keplerian equations of motion must give only a rough approximation, while the action of the tides must contribute its part to mask the influence of the

t	O_1	C_1	O_1-C_1	O_2	C_2	O_2-C_2
—72h	11.96	11.97	—0.01	11.96	11 97	—0.01
—66	11.88	11.87	+ .01	11.87	11.88	— .01
—60	11.72	11.67	+ .05	11.62	11.71	— .09
—54	11.53	11.37	+ .16	11.33	11.46	— .13
—48	11.26	10.95	+ .31	10.89	11.15	— .26
—42	10.87	10.37	+ .50	10.42	10.75	— .33
—36	10.23	9.61	+ .62	9.92	10.27	— .35
—30	9.06	8.60	+ .46	9.38	9 72	— .34
—24	7.17	7.27	— .10	8 86	9.10	— .24
—18	5.15	5.53	— .38	8.34	8.43	— .09
—12	3.80	3.53	+ .27	7.92	7.82	+ .10
— 6	3.20	3.09	+ .11	7.61	7.57	+ .04
0	3.00	3.00	.00	7.50	7.50	.00
+ 6	3.16	3.09	+ .07	7.75	7.57	+ .18
+12	3.77	3.53	+ .24	8.11	7.82	+ .29
+18	5.07	5.53	— .46	8.68	8.43	+ .25
+24	6.75	7.27	— .52	9.26	9.10	+ .16
+30	8.36	8.60	— .24	9.80	9.72	+ .08
+36	9.40	9.61	— .21	10.30	10.27	+ .03
+42	10.07	10.37	— .30	10.75	10.75	.00
+48	10.62	10.95	— .33	11.08	11.15	— .07
+54	11.13	11.37	— .24	11.42	11.46	— .04
+60	11.52	11.67	— .15	11.61	11.71	— .10
+66	11.78	11.87	— .09	11.87	11.88	— .01
+72	11.93	11.97	— .04	11.94	11.97	— .03

excentricity on the course of the light-curve. We conclude that, from the light-curves we can only infer that the orbit is nearly circular.

At all events there is no reason to assume an increase or decrease of the, certainly very small, excentricity. A comparison of the elements a and q might lead us to conjecture that the distance of the two celestial bodies has diminished since the time of ARGELANDER. The

increase in q is in agreement with such a supposition, but the continual lengthening of the period seems to clash with it.

15. *Computation of the orbit from the spectrographs of* BELOPOLSKY. In the computation of the orbit from the velocities in the line of sight as derived by B. from the displacement of the bright *F*-line in the spectrographs of 1892, the method of WILSING [1]) has been adopted. For very small excentricities it is to be preferred to that of LEHMAN-FILHÉS.

The first column contains the mean time of observation at PULKOWA; the 2nd gives the phase in the light-period of 12.91 days. We have assumed, in accordance with the formula of ARGELANDER, as corrected by PANNEKOEK, that the principal minimum occurred on 1892 Sept 25. 781 M. T. Greenwich (= 25d.865 M. T. PULKOWA).

The 3rd column contains the velocities, expressed in geographical miles, reduced to the sun. They have been taken, with slight modifications, from the Memorie della Soc. d. Spettr. It., vol. XXII. For BELOPOLSKY has applied a constant correction — 2.1 G.M. for the velocity of the earth, whereas in reality this velocity varies between — 1.6 and — 2.3 G.M.

T	Phase	Veloc. in G.M.	V_r	O-C
	d			
Sept. 23.3	10.34	—11.2	—11.25	+0.05
24.4	11.44	—11.6	—10.09	—1.51
25.4	12.44	— 4.4	— 2.58	+1.82
27.3	1.44	+ 4.8	+ 5.28	—0.48
30.3	4.44	+10.7	+10.50	+0.20
Oct. 2.3	6.44	+ 1 7	+ 2.09	—0.39
3.3	7.44	— 3.6	— 2.71	—0.89
7.3	11.44	— 9.5	—10.09	+0.59
11.3	2.53	+10.1	+10.29	—0.19
19.3	10.53	—12.4	—11.29	—1.11
20.3	11.53	—10.3	— 9.84	—0.46
26.3	4.62	+10.6	+ 9.98	+0.62
Nov. 25.2	8.70	— 6.7	— 7.85	+1.15
26.2	9.70	—10.2	—10.62	+0 42

[1]) Dr. J. WILSING. Ueber die Bestimmung von Bahnelementen enger Doppelsterne aus spectroskopischen Messungen der Geschwindigkeitscomponenten. A. N. no. 3198.

With the notations of WILSING these observations lead to the following normal equations:

$$+ 14\,g_0 - 3.17\,a_1 + 1.56\,b_1 - 3.41\,a_2 - 1.28\,b_2 = -42.00$$
$$- 3.17\,g_0 + 7.64\,a_1 - 1.68\,b_1 + 1.81\,a_2 + 1.02\,b_2 = +90.78$$
$$+ 1.56\,g_0 - 1.68\,a_1 + 6.36\,b_1 - 2.71\,a_2 - 0.27\,b_2 = -37.81$$
$$- 3.41\,g_0 + 1.81\,a_1 - 2.17\,b_1 + 8.52\,a_2 + 0.32\,b_2 = +30.74$$
$$- 1.28\,g_0 + 1.02\,a_1 - 0.27\,b_1 + 0.32\,a_2 + 5.48\,b_2 = +8.57$$

Solution:

$g_0 = -0.097$ G.M. = constant velocity towards the sun.

$a_1 = -\,an\sin i \sin(\omega' + M_0) = +11.196$; $b_1 = an\sin i \cos(\omega' + M_0) = -2.953$

$a_2 = -\,ean\sin i \sin(\omega' + 2M_0) = +0.498$; $b_2 = ean\sin i \cos(\omega' + 2M_0) = -0.708$

ω' is the longitude of the periastron, counted from ☊; M_0 the mean anomaly at the beginning of the light-period, consequently:

$$an\sin i = 11.579;\ \omega' = 115°20';\ M_0 = 139°54';\ e = 0.075.$$

As $\omega' + M_0 = 180° + 75°12'$, the elements belong to the body which, during the principal minimum, eclipses the other. Conjunction takes place, when $\omega' + v = 270°$, i.e. $0^d.39$ days after the time of the principal minimum, as computed from the empirical formula.

In the following table the 4th column shows the computed velocities, the 6th the outstanding deviations.

16. *Spectrographs of* 1897.

The velocities (in G. M), derived by *B.* from the displacements of the dark *Mg*-line $\lambda = 448.16\,\mu\mu$, have been taken unchanged from the Memorie della Soc. degli Spettr. It. vol XXVI. The empirical formula leads to the epoch 1897 June 22 16h. 24 M. T. PULKOWA for the principal minimum.

In a first approximation I determined a circular orbit and found:

$$g_0 = -2.094;\ an\sin i = 24.210;\ \omega' + M_0 = 89°.30'.1.$$

Afterwards corrections were derived by the aid of the formula:

$$d\frac{dz}{dt} = dg_0 + KndT\sin(\omega' + v) + dK \cdot \cos(\omega' + v) +$$
$$+ Ke\cos\omega'\cos 2(\omega' + v) + Ke\sin\omega'\sin 2(\omega' + v).$$

in which $K = an\sin i$, $T =$ time of periastron-passage.

This formula is obtained from the general differential-formula [1]), by putting $d\mu = 0$, $d\omega = 0$ and by further neglecting 2nd and higher powers of e. We thus find the following normal equations:

[1]) Vide: BAUSCHINGER, Die Bahnbestimmung der Himmelskörper, N°. 199.

$$+\ 26dg_0 - 1.35K\mu dt - 2.01dK - 2.22Ke\cos\omega' - 2.03Ke\sin\omega' = -0.05$$
$$-1.35\ ,,\ +13.18\ ,,\ -\ 1.02\ ,,\ +\ 1.31\ ,,\ +\ 0.16\ ,,\ =+6.17$$
$$-2.01\ ,,\ -\ 1.02\ ,,\ +12.82\ ,,\ -\ 0.28\ ,,\ +\ 0.26\ ,,\ =\ 0.00$$
$$-2.22\ ,,\ +\ 1.31\ ,,\ -\ 0.28\ ,,\ +13.88\ ,,\ -\ 0.31\ ,,\ =+4.75$$
$$-2.03\ ,,\ +\ 0.16\ ,,\ +\ 0.26\ ,,\ -\ 0.31\ ,,\ +12.12\ ,,\ =-5.98$$

	T	Phase	Veloc. in G.M.	V_r	$Q-C$
	h	d h			
June 20	11.5	10 17.0	+18.27	+19.29	—1.02
22	12.0	12 17.6	— 2.60	+ 0.28	—2.88
23	12.4	0 20.2	—10.62	—10.98	+0.36
24	12.1	1 19.9	—20.40*)	—19.92	—0.48
28	11.6	5 19.4	—11.14	—10.77	—0.37
30	11.1	7 18.9	+14.16	+12.41	+1.75
July 2	11.9	9 19.7	+21.38	+22.42	—1.04
8	12.3	2 22.3	—24.97*)	—25.56	+0.59
9	11.4	3 21.4	—25.68	—25.32	—0.36
10	11.1	4 21.1	—21.27	—19.87	—1.40
11	11.0	5 21.0	— 8.83	—10.00	+1.17
12	11.5	6 21.5	+ 3.24	+ 2.34	+0.90
13	11.4	7 21.4	+13.15	+13.43	—0.28
15	11.4	9 21.4	+24.15	+22.34	+1.81
17	11.2	11 21.2	+10.34	+ 9.35	+0.99
21	11.2	2 23.4	—27.52	—25.65	—1.87
22	11.2	3 23.4	—23.48	—25.07	+1.59
24	10.3	5 22.3	— 9.28	— 9.37	+0.09
25	10.2	6 22.2	+ 0.53	+ 2.70	—2.17
26	10.0	7 22.0	+12.77	+13.68	—0.91
27	10.2	8 22.0	+21.03	+20.86	+0.17
30	10.1	11 22.1	+10.11	+ 9.16	+0.95
31	10.2	0 0.6	— 1.03	— 2.01	+0.98
Aug. 2	9.7	2 0.1	—20.36	—21.64	+1.28

*) Mean of two observations.

Solution:

$$dg_0 = +0.0124 \quad ; \quad K\mu dT = +0.450 \quad ; \quad dK = +0.054 ;$$

$$Ke \cos \omega' = +0.292 \quad ; \quad Ke \sin \omega' = -0.489.$$

from which we get the elements:

$$g_0 = -2.082\ GM. \quad ; \quad an \sin i = 24.264 \quad ; \quad e = 0.0235 ;$$

$$\omega' = 300°51' \quad ; \quad \omega' + M_0 = 88°26' ;$$

whereas the conjunction coincides perfectly with the principal minimum, *the difference in time amounting to less than* 0.01 *days.* Evidently this is the principal minimum. This is in accordance with the fact that the difference of the longitudes found for the two periastra deviates but slightly from 180° (300°51′ — 115°20′). This may be partly due to a fortunate coincidence.

Meanwhile the excentricity of the 2nd orbit is more than three times *smaller* than that of the 1st, while the velocity in the direction towards the sun found for the whole system is 2 Geogr. miles greater in the 2nd case.

If the latter difference is real, this acceleration would have caused a *shortening* of the period between 1892 and 1897. As, however the measures of 1892, according to the judgment of Prof. H. C. Vogel "nicht als ganz einwurfsfrei angesehen werden können" [1]), we suspend our judgment to the time that Prof. Belopolsky will again have taken up his beautiful investigations on the spectrum of β Lyrae, particularly about the F- line. Already in 1897 he communicated his intention to do so.

If we put $i = 90°$, the semiaxes major are:

$$a_1 = 2056000\ G.M. \quad ; \quad a_2 = 4307000\ G.M.$$

From Kepler's third law we derive therefrom, roughly

$$m_1 = 17.1 \text{ sun's masses} \quad ; \quad m_2 = 8.1 \text{ sun's masses.}$$

17. In our opinion the preceding considerations justify the conclusion that the data about β Lyrae do not furnish a sufficient basis for a decision about any change in the elements, in particular in the excentricity. For the rest, owing to our ignorance on the circumstances in such a close system, the adopted explanation of the light variation can only claim to give a rough approximation — a rude imitation of a very complicated mechanism.

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[1]) Ueber das Spectrum von β Lyrae. Sitzungsb. Ak. Berlin. 1894 VI.

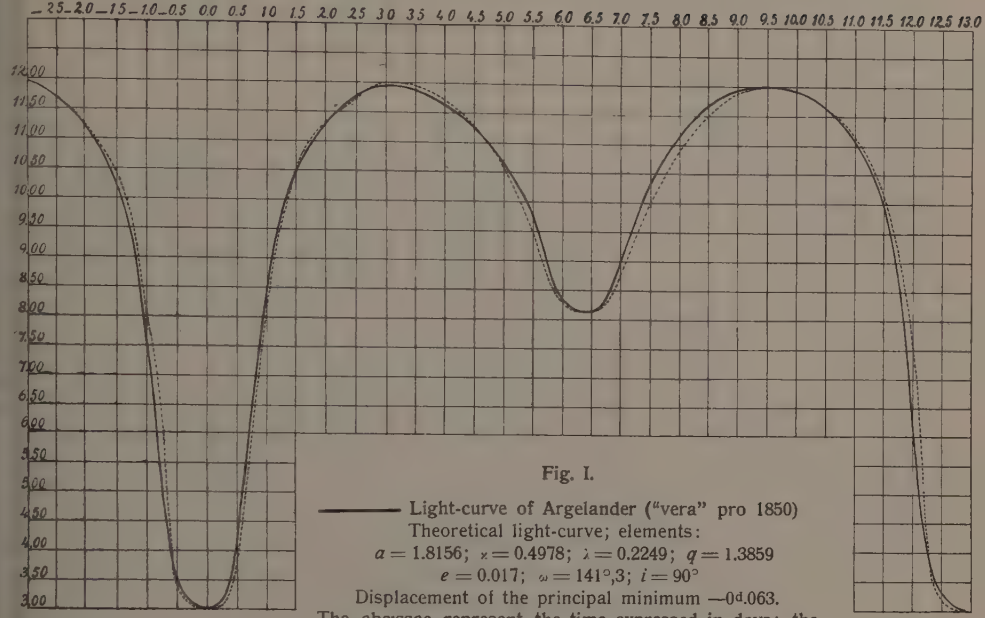

Fig. I.

——— Light-curve of Argelander ("vera" pro 1850)
Theoretical light-curve; elements:
$a = 1.8156$; $\varkappa = 0.4978$; $\lambda = 0.2249$; $q = 1.3859$
$e = 0.017$; $\omega = 141°,3$; $i = 90°$
Displacement of the principal minimum $-0^d.063$.
The abcissae represent the time expressed in days; the ordinates the light-intensity in grades.

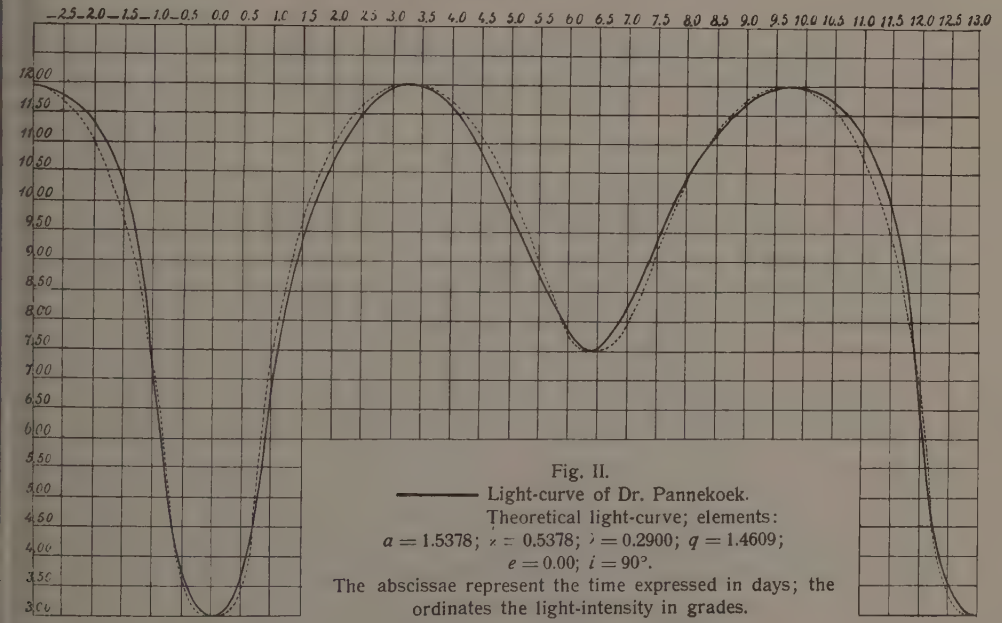

Fig. II.

——— Light-curve of Dr. Pannekoek.
Theoretical light-curve; elements:
$a = 1.5378$; $\varkappa = 0.5378$; $\lambda = 0.2900$; $q = 1.4609$;
$e = 0.00$; $i = 90°$.
The abscissae represent the time expressed in days; the ordinates the light-intensity in grades.

Mathematics. — "*The section of the measure-polytope M_n of space Sp_n with a central space Sp_{n-1} perpendicular to a diagonal.*" By Prof. P. H. SCHOUTE.

(Communicated in the meeting of December 28, 1907).

We determine the indicated section in three different ways:

1. by means of the projection of M_n on the diagonal,
2. with the aid of the projection of M_n on a plane through two opposite edges intersecting the diagonal,
3. by regarding regular simplexes.

I. *The projection of M_n on a diagonal.*

1. We can easily prove both analytically and synthetically the following theorem:

"The vertices of the measure-polytope M_n project themselves on a "diagonal in $n+1$ points, namely in the ends of the diagonal and "in the $n-1$ points, which divide the latter into n equal parts; in "these $n+1$ points are projected successively

$$1, n, \tfrac{1}{2}n(n-1), \ldots . \tfrac{1}{2}n(n-1), n, 1$$

"points, where these numbers are the coefficients of the terms "of $(a+b)^n$".

From this general theorem ensue the results for $n = 4, 5, 6, 7, 8$ given in the diagrams added here (see the expanding plate). An explanation of the sketch belonging to $n = 4$ will sufficiently explain the others.

The horizontal lines of this figure always represent the same diagonal on which the projection takes place; on these ten lines are successively indicated the projections of vertices, of edges, of faces and of bounding bodies In order to find space for the figures indicating the numbers, the thick projection-lines have been broken off, where such was necessary.

If we designate the five points on the diagonal by a, b, c, d, e, — see the bottom line of the ten horizontal ones — then in these places — see the topmost of the ten lines — 1, 4, 6, 4, 1 vertices are projected there — bear in mind $(1+1)^4$.

On the four equal segments ab, bc, cd, de are projected successively 4, 12, 12, 4 edges — think of $4(1+1)^3$.

In like manner the three equal segments ac, bd, ce are successively the projections of 6, 12, 6 faces — think of $6(1+1)^2$.

Finally on the two equal segments ad, be are projected successively 4, 4 bounding bodies — think of $4(1+1)$.

It is easy to deduce from this the results given in the other diagrams for $n = 5, 6, 7, 8$, if we keep in mind, that the coefficients by which $(1+1)^4$, $(1+1)^3$, $(1+1)^2$, $(1+1)$ are multiplied are 1, 4, 6, 4 and so by addition of unity at the end pass into a repetition of $(1+1)^4$.

2. More generally holds the following theorem, comprising the preceding:

"The vertices of each bounding M_p of M_n $(p \leqq n)$ are projected on "the diagonal of M_n in $p+1$ successive points of division of that "diagonal; here again the projections are distributed according to "the coefficients $1, p, \frac{1}{2}p(p-1), \ldots$ of $(a+b)^p$ over these $p+1$ "successive points."

The vertices of a bounding square are projected in three of the $n+1$ points, which naturally demands the division 1, 2, 1. The vertices of a bounding cube are projected in four of the $n+1$ points, which of necessity must lead to the division 1, 3, 3, 1 as by the preceding the division 2, 2, 2, 2 is excepted.

From this ensues then directly the following theorem:

"The section of a space Sp_{n-1} perpendicular to the diagonal of M_n "forming the axis of projection, with the space Sp_p bearing a bounding "M_p of M_n is an Sp_{p-1} in Sp_p perpendicular to the diagonal of "M_p connecting the two vertices of M_p projecting themselves in the "ends of the projection of M_p." [1])

But there is more. If p' (M_p) represents the section of a measure-polytope M_p with a space Sp_{p-1} of its space Sp_p perpendicular to one of its diagonals in a point of which the distance to the centre of the diagonal in the diagonal as unity amounts to $\frac{1}{2} - p'$, from which is evident that $p' \leqq \frac{1}{2}$, the two theorems hold:

"For even n a bounding measure-polytope M_p of M_n is intersected "by the central space Sp_{n-1} perpendicular to the diagonal of M_n

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[1]) The indicated diagonal d_p of M_p is the projection of the axis of projection d on the space Sp_p of M_p; so we can obtain the projections of the vertices of M_p on d by projecting these vertices first in Sp_p on d_p and projecting afterwards on d the points found on d_p by the preceding means.

As d_p and d in the edge of M_n as unity are represented by $\sqrt{p}$ and $\sqrt{n}$ and d_p is projected on d as $\frac{p}{n}$ of d, the cosine of the angle between d and Sp_p is equal to $\frac{1}{n}\sqrt{np}$.

"according to an $\frac{a}{p}(M_p)$, where a according to circumstances can "assume for even p one of the $\frac{p}{2}$ values $1, 2, \ldots \frac{p}{2}$, for odd p one "of the $\frac{p-1}{2}$ values $1, 2, \ldots \frac{p-1}{2}$."

"For odd n the measure-polytope M_p is intersected under the same "circumstances according to a $\frac{2a-1}{p}(M_p)$ where a can assume for "even p one of the $\frac{p}{2}$ values $1, 2, \ldots \frac{p}{2}$, for odd p one of the $\frac{p+1}{2}$ "values $1, 2, \ldots \frac{p+1}{2}$."

We shall now, instead of losing ourselves in further generalities, give the full results of the diagrams for the cases $n = 4, 5, 6, 7, 8$ to make clear the above. In order to be able to indicate easily ratios of measure we shall suppose the edge of M_n to be unity of length.

3. Case $n = 4$. The space — see first diagram — perpendicular in the centre c of diagonal ae to this diagonal contains the six vertices of M_4 projecting themselves in c and cuts — see lines 3 and 4 — no edge; so the section has six vertices. This same space cuts twelve faces — see line 7 — according to $\frac{1}{2}(M_3)$ and eight bounding bodies — see lines 9 and 10 — according to $\frac{1}{3}(M_3)$; so the section has twelve edges with a length $\sqrt{2}$ and eight equilateral triangles as faces. So the section is a $(6, 12, 8)$ and, indeed, the regular octahedron with edges $\sqrt{2}$.

Case $n = 5$. We find — see second diagram — thirty vertices generated by intersection of edges, sixty edges, forty faces and ten bounding bodies, so a $(30, 60, 40, 10)$. The vertices are of the same kind, the edges have as $\frac{1}{4}(M_3)$ the length $\frac{1}{2}\sqrt{2}$. The forty faces consist of twenty $\frac{1}{2}(M_3)$ and two times ten $\frac{1}{6}(M_3)$, i. e. of twenty hexagons and twenty triangles, both regular [1]) with sides $\frac{1}{2}\sqrt{2}$.

[1]) Where the regularity is obvious — as e. g. with the triangles by the equal length of all edges, etc. — the additional "equilateral" or "regular" will in future be left out.

Each of the ten bounding bodies is as $\frac{3}{8}(M_4)$ — compare in the first diagram the section with a space perpendicular to *ae* in the point in the middle between *c* and *d* — a (12, 18, 8) bounded by four $\frac{1}{2}(M_3)$ and four $\frac{1}{6}(M_3)$, i. e. by four of the hexagons and four of the triangles, and therefore a tetrahedron truncated regularly at the vertices, i. e. the first of the equiangular semi-regular (Archimedian) bodies.

Case $n = 6$. Out of the third of the diagrams we read that the section is a (20, 90, 120, 60, 12). All the edges have a length $\sqrt{2}$, all the faces are triangles. The bounding bodies are for one half (30) as $\frac{1}{2}(M_4)$ octahedra, for the other half (15 + 15) as $\frac{1}{4}(M_4)$ tetrahedra. The twelve bounding polytopes are as $\frac{2}{5}(M_5)$ — compare now again the second diagram — polytopes (10, 30, 30, 10) bounded by five of the octahedra and five of the tetrahedra, which can be regarded as regular five-cells, regularly truncated at the vertices as far as half of the edges, so as to lose all the original edges by this truncation.

Case $n = 7$. We arrive at a (140, 420, 490, 280, 84, 14). The length of the edges is $\frac{1}{2}\sqrt{2}$. The 490 faces consist of 210 hexagons and 280 triangles, the 280 bounding bodies of 210 truncated tetrahedra and 70 tetrahedra, the 84 four-dimensional bounding polytopes of 42 polytopes $\frac{1}{2}(M_5) = (30, 60, 40, 10)$ found already under $n = 5$ and 42 polytopes $\frac{3}{10}(M_5) = (20, 40, 30, 10)$ bounded by five truncated tetrahedra and five tetrahedra — regular five-cells truncated at the vertices as far as a third of the edges. The 14 five-dimensional bounding polytopes are as $\frac{5}{12}(M_6)$ polytopes (60, 150, 140, 60, 12) bounded by six (30, 60, 40, 10) and six (20, 40, 30, 10).

Case $n = 8$. Here a (70, 560, 1120, 980, 448, 112, 16) is the result. The length of the edges is $\sqrt{2}$, all faces are triangles. The

980 bounding bodies consist of 420 octahedra and 560 tetrahedra the 448 four-dimensional bounding polytopes of 336 polytopes $\frac{2}{5}(M_5)$ and 112 polytopes $\frac{1}{5}(M_5)$, i. e. of 336 five-cells truncated as far as half of the edges, found under $n = 6$, and 112 five-cells. The 112 five-dimensional bounding polytopes are as far as one half is concerned $\frac{1}{2}(M_6) = (20, 90, 120, 60, 12)$ already found above, as far as the other half is concerned $\frac{1}{3}(M_6) = (15, 60, 80, 45, 12)$ bounded by six five-cells truncated as far as half the length of the edges and six five-cells. Finally the sixteen six-dimensional bounding polytopes are as $\frac{3}{7}(M_7)$ polytopes (35, 210, 350, 245, 54, 84) bounded by seven (20, 90, 120, 60, 12) and seven (15, 60, 80,45, 12) [1]).

From this all we easily deduce the following general laws:

"The vertices of the section are vertices of M_n for even n, for odd n they are centres of edges of M_n; they are always of the same kind [2])."

"The common length of the edges is $\sqrt{2}$ for even n and $\frac{1}{2}\sqrt{2}$ for odd n; they are always of the same kind [3])."

"The faces are triangles for even n, hexagons and (smaller) triangles [4]) for odd n."

"The bounding bodies are octahedra and tetrahedra for even n, truncated tetrahedra and (smaller) tetrahedra for odd n".

"The four-dimensional bounding polyhedra are five-cells truncated as far as halfway the edges and five-cells for even n, five-cells

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[1]) If we had set to work, when enumerating the results, in that sense inversely that with each new value of n of the bounding polytopes with the greatest number of dimensions we had descended to the vertices, we should have furnished a geometrical variation of the well known nursery-book: "the house that Jack built". However with two differences. When descending from every one round higher of the ladder we pass *every other time* again the same stadia and the ladder is a Jacob's ladder with an infinite number of rounds.

[2]) That is, in each vertex as many edges meet in the same way, etc.

[3]) The cases $n =$ odd seem to be an exception to this, as there are for the truncated tetrahedra two kinds of edges, namely: sections of two hexagonal faces and sections of an hexagonal and a triangular face. However, this is only apparently. For, for each edge we find that in the section itself always again the number of faces passing through it of each of the two sorts is steadfast, thus for $n = 5$ two hexagonal faces and one triangular one.

[4]) We do not mention here, that for $n = 3$ only an hexagon appears. Neither that of the bounding bodies the tetrahedra do not appear for $n = 4$, etc.

truncated as far as a third of the edges and (smaller) five-cells for odd n."

Etc., etc. [1]).

The above results are for the greater part given in the general theorems mentioned above.

II. The projection of M_n on a plane through two opposite edges cutting the diagonal.

4. For each value of n the indicated projection — see fig. 1 for $n = 8$ and $n = 9$ — is a rectangle $PQQ'P'$ with the sides 1 and

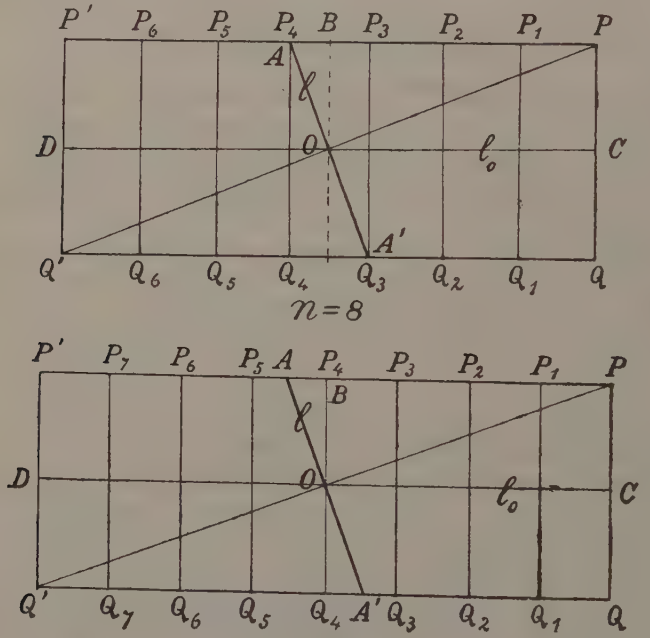


Fig. 1.

$\sqrt{n-1}$, which is divided by $n-2$ lines $P_1Q_1, P_2Q_2, \ldots P_{n-2}Q_{n-2}$ parallel to the shorter sides PQ, $P'Q'$ into $n-1$ equal rectangles.[2])

[1]) We break off here because not until the third division do we indicate that everything making its appearance in the section can be regarded as simplex or truncated simplex.

The symbol which indicates the numbers of vertices, edges, faces, etc. for arbitrary n is purposely omitted as its form is rather complicated.

[2]) To compare the treatise "On the sections of a block of eightcells, etc." (Verhandelingen der K. A. v. W., vol IX, N. 7).

The diagonal on which the intersecting space Sp_{n-1} is at right angles is one of the diagonals of the rectangle, e.g, PQ'. If the normal erected in the centre O of PQ' on this line, representing the projection of the intersecting space Sp_{n-1}, cuts the side PP' in A, this point A always lies at a distance $\frac{1}{2\sqrt{n-1}}$ from the centre B of PP'. For in the right-angled triangle AOP we find that B is the foot of the normal let down out of O on AB and from this ensues $AB.BP=OB^2$ and therefore $AB=\frac{1}{4} : \frac{1}{2}\sqrt{n-1}$. So A coincides for even n with the point of division $P_{\frac{n}{2}}$ and this point lies for odd n in the middle between $P_{\frac{n-1}{2}}$ and $P_{\frac{n+1}{2}}$. From this it is again evident that the vertices of the section are vertices of M_n for even n and centres of edges of M_n for odd n.

In the paper quoted above which restricts itself to the case $n=4$ we find in a note how we can regard the section under observation as a "rhombotope" truncated at both sides; the course of thoughts is as follows. Let us imagine in the direction of the edges $PQ, P'Q'$ on either side an infinite number of measure-polytopes M_n piled on each other and let us then remove the measure-polytopes M_{n-1}, projecting themselves on PP', QQ' and lines parallel to these, with which the successive polytopes M_n bound each other; then a prism is formed with M_{n-1} as right section. If this prism is intersected by a space Sp_{n-1} which projects itself along the perpendicular l_0 let down out of O on PQ, the section is thus an M_{n-1}. What variation does this section M_{n-1} of the prism undergo when we substitute for the intersecting space projecting itself along l_0 an other one which projects itself along a line l_φ through O, enclosing with l_0 an angle φ? As is easy to see from the figure this variation consists of a regular enlargement of the perpendiculars let down out of the boundary of M_{n-1} on the space Sp_{n-2}, projecting itself in O, which enlargement means a multiplication of those perpendiculars by $sec\,\varphi$ and can be regarded as a *stretching* in the direction of the diagonal CD. As for $n=4$, where M_{n-1} becomes a cube, such a stretching makes a rhombohedron of a cube, out of M_{n-1} is formed in general what we call a rhombotope.

Just as the rhombohedron regarded as a whole passes into itself when it is revolved 120° about the axis, or — in other words — just as the axis of the rhombohedron has a period three, the axis of the rhombotope under consideration has a period $n-1$. Let us

now imagine this rhombotope, for the special case that the projection of the intersecting space Sp_{-1} — so also the projection of the rhombotope itself — falls along OA and let us truncate it by the two spaces Sp_{n-2} standing normal to the plane of projection in the ends A, A' of the segment AA' of that projection lying inside the rectangle and cutting the axis of the rhombotope therefore at right angles; we then find the required section, to be indicated according to the number of its dimensions by D_{n-1}. We directly determine the length of the axis of the untruncated rhombotope and of D_{n-1}, but before this we shall deduce some general theorems easy to find.

5. The edges of M_n project themselves on the assumed plane *either* along one of the n lines $PQ, P_1Q_1, P_2Q_2, \ldots P_{n-2}Q_{n-2}, P'Q'$, *or* as parts of PP' or QQ'. Because the vertices of D_{n-1} must be vertices of M_n or points of intersection with edges of M_n, these points project themselves — compare fig. 1 for $n = 8$ and for $n = 9$ — for even n exclusively in the ends A, A', for odd n exclusively in those ends and in the centre O.

From this ensues for $n = 2n'$ the general theorem:

"The section $D_{2n'-1}$ of $M_{2n'}$ is a $2n' - 1$-dimensional prismoid with respect to each pair of opposite bounding spaces $Sp_{2n'-2}$ and so in $2n'$ ways".

Here follow two theorems holding for arbitrary n:

"Each line through the centre O normal to two opposite bounding spaces Sp_{n-2} is axis of D_{n-1} with the period $n-1$."

"Each space Sp_{n-2} through O parallel to a bounding space Sp_{n-2} divides D_{n-1} into two congruent $n-1$-dimensional prismoids."

In the demonstration of these three theorems the entire equivalence of a pair of opposite bounding spaces Sp_{n-2} with any other pair has the chief part; moreover the third causes us to inquire how the space Sp_{n-2} through the centre parallel to a bounding space intersects D_{n-1}. We prove as follows that this section is a D_{n-2}.

If the projection l of the intersecting space Sp_{n-1} revolves round O, the $Sp^{(0)}_{n-2}$ normal to the plane of projection in O remains in its place and Sp_{n-1} thus describes a pencil with this $Sp^{(0)}_{n-2}$ as axial space. Therefore then the varying section keeps going through the section of $Sp^{(0)}_{n-2}$ with M_n. We can easily know the nature of this section of $n-2$ dimensions by regarding the case in which l coincides with l_0. Then our D_{n-1} is an M_{n-1} and this measure-polytope projecting itself along l_0 is intersected according to a D_{n-2} by the space $Sp^{(0)}_{n-2}$, which is in O normal to the plane of projection and

which therefore bisects the diagonal CD of this M_{n-1}. This D_{n-2} is the section of D_{n-1} with the space $Sp^{(0)}_{n-2}$, through O parallel to the spaces $Sp^{(0)}_{n-2}$, which are in A_1 and A' normal to the axis and which truncate the rhombotope. So we find:

"Each space $Sp^{(0)}_{n-2}$ through the centre O parallel to a bounding space Sp_{n-2} intersects D_{n-1} according to a D_{n-2} of which O is again the centre."

From this follows again more generally:

"Each space $Sp^{(0)}_p$ $(0 < p < n-1)$ through the centre O parallel to a bounding space Sp_p intersects D_n according to a D_{p-1}, of which O is again the centre".

Thus we find ascending from below:

"Each chord of D_{n-1} through O parallel to an edge has a length $\sqrt{2}$, each plane through O parallel to a face intersects D_{n-1} according to a regular hexagon with sides $\frac{1}{2}\sqrt{2}$, each space through O parallel to a bounding body intersects D_{n-1} according to a regular octahedron with edges $\sqrt{2}$, etc."

6. We retrace our steps and determine of the above mentioned rhombotope the length of the axis before and after the truncation. Out of the similitude of the triangles AOB and POC follows in connection with the length $\frac{1}{2}\sqrt{n-1}$, $\frac{1}{2}\sqrt{n}$, $\frac{1}{2}$ of OC, OP, OB for OA the value $\frac{1}{2(n-1)}\sqrt{n(n-1)}$ and so for half of the unmutilated axis which is n—1 times as large $\frac{1}{2}\sqrt{n(n-1)}$. If we represent by $Rh_p[q, r]$ a rhombotope with p dimensions of which q is the length of the axis, r are the parts of the axis removed by the truncation, the section D_{n-1} has to be represented by the symbol $Rh_{n-1}\left[\sqrt{n(n-1)}, \frac{n-2}{2(n-1)}\right]$

So the theorem holds:

"We obtain the section D_{n-1}, if we allow the measure-polytope M_{n-1} to pass in the indicated way by stretching in the direction of a diagonal as far as $\sqrt{n}$ times the original length into a rhombotope with a length of axis $\sqrt{n(n-1)}$ and if we truncate this rhombotope by two spaces Sp_{n-2} normal to the axis to a

$$Rh_{n-1}\left[\sqrt{n\,(n-1)}, \frac{n-2}{2\,(n-1)}\right]"^{1})$$

III. Explanation in details of the connection of D_{n-1} with regular and regularly truncated simplexes.

7. We consider in the space Sp_n a rectangular system of coordinates with an arbitrary point O as origin and OX_1, OX_2,... OX_n as axes, and we now call the $2^{n\,\text{th}}$ part of that space which is the locus of the point with only positive coordinates the "n-edge $O\,(X_1\,X_2 \ldots X_n)$".

If A, A' are two opposite vertices of a measure-polytope M_n of Sp_n and if $AA_1, AA_2, \ldots AA_n$ are the edges passing through A and $A'A'_1, A'A'_2, \ldots A'A'_n$ the edges parallel to these but directed oppositely, then M_n can be regarded as the part of the space Sp_n common to the two n-edges $A\,(A_1\,A_2 \ldots A_n)$ and $A'\,(A'_1\,A'_2 \ldots A'_n)$.

If we intersect this figure of the two oppositely orientated n-edges and the measure-polytope M_n common to both by an arbitrary space Sp_{n-1}, the two n-edges are intersected along two oppositely orientated simplexes and the section of M_n with that space Sp_{n-1} appears as the part of that space that is enclosed at the same time by both simplexes situated in that space. If the selected space is normal to the diagonal AA', connecting the vertices of the n-edges, the simplexes are regular and they have the point of intersection P of the intersecting space Sp_{n-1} with AA' as common centre of gravity. So the general theorem holds:

"The section of M_n with a space Sp_{n-1} normal to a diagonal can always be regarded as a part of that space Sp_{n-1} enclosed by two definite concentric, oppositely orientated, regular simplexes of that space".

If we wish to make use of this theorem we must determine in a more detailed way the length of the edges of those oppositely orientated regular simplexes with common centre of gravity.

8. If we think the intersecting space Sp_{n-1} to be normal to the

[1]) This theorem shows distinctly why the sections of an octahedron parallel to two faces must be identical to those of a cube by planes normal to a diagonal in points of the middle third part of that line. The same in other words: If we truncate a cube with the unity of edge at two opposite vertices by planes normal to the connecting line in the points dividing this diagonal into three equal parts and if we compress an octahedron with edges $\sqrt{2}$ in the direction of the normal on two parallel faces as far as half the thickness, then we cause the same solid to be generated in two different ways.

diagonal AA' in the first point of division A_1, at a distance $\frac{1}{n}\sqrt{n}$ from A, the section is a simplex with edge $\sqrt{2}$. So the two simplexes, generated when an arbitrary point P of AA' is substituted for point A_1, have edges of a length of $AP\sqrt{2n}$ and $A'P\sqrt{2n}$, wherefore we indicate them, also with reference to the number of vertices, by $S_n(AP\sqrt{2n})$ and $S'_n(A'P\sqrt{2n})$. So the theorem holds:

"If we shove an M_n, of which the diagonal AA' is normal to a given space Sp_{n-1}, in the direction of that diagonal through that space Sp_{n-1}, so that the spaces Sp_{n-1} of the bounding polytopes M_{n-1} move parallel to themselves, the section of Sp_{n-1} with the moving polytope M_n is at every moment the part of that space Sp_{n-1} that is enclosed within two concentric, yet oppositely orientated, regular simplexes $S_n(p\sqrt{2n})$ and $S'_n(p'\sqrt{2n})$ where p and p' are connected in such a way that the sum $p+p'$ is equal to $\sqrt{n}$. During that movement of M_n the common centre of gravity of the two simplexes remains in its place and the spaces Sp_{n-2} of the bounding simplexes S_{n-1} and S'_{n-1} move parallel to themselves; whilst simplex S_n expands itself from this common centre of gravity to a simplex $S_n(n\sqrt{2})$, simplex S'_n inversely contracts from a simplex $S'_n(n\sqrt{2})$ to this point".

At the moment when this process has got halfway and the two simplexes are of the same size we find:

"The section D_{n-1} is the part of the intersecting space Sp_{n-1} enclosed by two definite equal concentric yet oppositely orientated regular simplexes $S_n\left(\frac{1}{2}n\sqrt{2}\right)$ and $S'_n\left(\frac{1}{2}n\sqrt{2}\right)$."

Thus for $n=3$ the regular hexagon with sides $\frac{1}{2}\sqrt{2}$ is the figure enclosed by two triangles with sides $\frac{3}{2}\sqrt{2}$ — think of the well-known trademark —, thus for $n=4$ the regular octahedron with edges $\sqrt{2}$ is the figure enclosed by two tetrahedra with edges $2\sqrt{2}$ — think of the two tetrahedra described in a cube and the octahedron common to both. So in general the problem in the space of n dimensions is reduced to another problem in space of $n-1$ dimensions and moreover the connection of the result with regular simplexes is explained.

If we think the simplex S_n to be white and the simplex S'_n to be black, the n bounding spaces Sp_{n-2} of D_{n-1} originating from S_n will be white, those originating from S'_n will be black. From this ensues that it must be possible to colour the $2n$ bounding spaces

Sp_{n-2} of D_{n-1} in such a way in turns white and black, that two opposite bounding spaces Sp_{n-2} have a different colour. The octahedron is really the only one of the regular bodies that allows this operation. [1])

9. If the simplex S_n expands from a point to an $S_n(n\sqrt{2})$ and at the same time S'_n contracts from an $S'_n(n\sqrt{2})$ to a point, then S_n lies at the beginning of the process within S'_n and at the end inversely S'_n lies within S_n. Gradually first the vertices, then the edges, then the faces, etc. of S_n have passed outward. We shall now investigate when that takes place.

From the diagrams of the expanding plate given in the first part it is evident, that the section of M_n with a space Sp_{n-1} changes its nature when the point of intersection P of that space Sp_{n-1} with the diagonal AA' passes one of the $n-1$ points of division $A_1, A_2, \ldots$ As the nature of the section of course also changes when bounding elements of S'_n lying inside S_n pass outward, the latter must take place at those moments when those points of division of the diagonal AA' of the moving M_n pass through the fixed space Sp_{n-1}. This theorem then really holds:

"In the translation of M_n in the direction of AA' through the space Sp_{n-1} in succession the vertices, the edges, the faces, bounding bodies, etc. of S_n come entirely outside S'_n at those moments that the point of intersection P of the diagonal AA' with the space Sp_{n-1} coincides successively with the points of division A_1, A_2, A_3, A_4, etc."

We regard — in order to prove this theorem — the arbitrary stadium of the simplexes $S_n(AP\sqrt{2n})$ and $S'_n(A'P\sqrt{2n})$, divide the n vertices of S_n in an arbitrary way into two groups β and γ of p and $n-p$ points, and indicate by β' and γ' the groups of the p and $n-p$ corresponding vertices of S'_n, by B, C, B', C' (fig. 2) the centres of gravity of the point-groups $\beta, \gamma, \beta', \gamma'$ — i.e. the

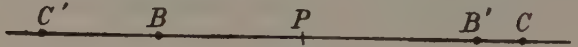


Fig. 2.

[1]) In contradiction to this seems that for $n=5$ through each edge *three* faces pass and thus *three* bounding bodies (12, 18, 8) lie around it. This contradiction however is only apparent; it is annulled by the remark that two bounding bodies (12, 18, 8) having a face in common agree or differ in colour according to the face being triangular or hexagonal. Of the three faces one is triangular, two are hexagonal; the bounding bodies to which the two hexagonal faces belong, differ in colour from the two others, these agreeing in colour.

centres of the bounding simplexes S_p, S_{n-p}, S'_p, S'_{n-p} with these points as vertices. Then the five points B, C, B', C', P lie in such a way upon the same right line, that B and C' lie on one side of P and B' and C on the other side, and we have

$$\left.\begin{array}{rr} p\ BP = (n-p)\ PC \\ (n-p)\ C'P = \quad p\ .\ PB' \end{array}\right\}, \quad \frac{AP}{PA'} = \frac{BP}{PB'} = \frac{CP}{PC'}.$$

We can now assert that the bounding simplex S_p of the vertices β of S_n lies entirely or partly inside S'_n when B is between C' and P, whilst S_p lies entirely outside S_n when C' lies between B and P. In other words: as AP increases, the bounding simplex S_p of S_n comes entirely outside S_n when B coincides with C' and the spaces Sp_{p-1} and Sp_{n-p-1} of S_p and S'_{n-p}, crossing each other in general entirely perpendicularly, become incident because they get the point $B = C'$, then common centre of gravity, as point of intersection.

Under the condition $BP = C'P$ follows from the equations

$$\frac{BP}{PC} = \frac{n-p}{p}, \quad \frac{PC}{C'P} = \frac{AP}{PA'}$$

the relation

$$(n-p)\ AP = p\ .\ PA',$$

which shows that P must coincide with the pth dividing point A_p of AA'.

10. If P coincides with A_p the spaces Sp_{p-1} and Sp_{n-p-1} of S_p and S'_{n-p} have, as we saw above, the common centre of S_p and S'_{n-p} in common. As this point of intersection of S_p and S'_{n-p} becomes vertex of the section, — if we call this again $\frac{p}{n}(M_n)$ in connection with preceding investigations — the theorem holds:

"The centres of the $\binom{n}{p}$ bounding simplexes S_p of a regular simplex $S_n(p\sqrt{2})$ form the vertices of a polytope congruent to $\frac{p}{n}(M_n)$ for $p = 1, 2, \ldots, n-1$."

For even $n = 2n'$ we have specially:

"The centres of the $\binom{2n'}{n'}$ bounding simplexes $S_{n'}$ of a regular simplex $S_{2n'}$ $(n'\sqrt{2})$ form the vertices of a $D_{2n'-1}$."

11. If P lies between A_p and A_{p+1} the vertices of the section of the two simplexes S_n and S'_n are furnished by the points of intersection of each bounding simplex S_{p+1} of S_n with the $p+1$

bounding simplexes S'_{n-p} of S'_n which have the property of counting among their $n-p$ vertices only one vertex corresponding to a vertex of this S_{p+1}; in each bounding simplex S_{p+1} these $p+1$ points of intersection form the vertices of a new regular simplex $\overline{S}_{p+1}$ which is concentric to the assumed one but oppositely orientated. We determine the length of the edges of this new simplex, for the definite case that P lies just in the middle between A_p and A_{p+1}, with the aid of reflections in quite close connection with the preceding.

If B, C, B', C' (fig. 3) are successively the centres of gravity of

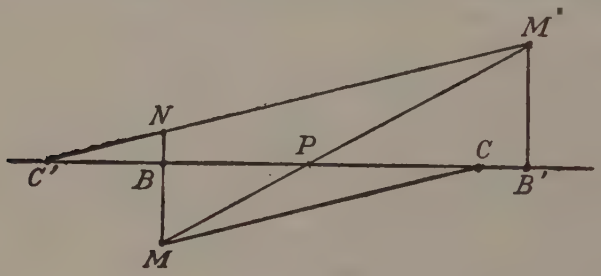


Fig. 3.

the bounding simplex S_{p+1}, of the bounding simplex S_{n-p-1} of the remaining vertices of S_n and of the bounding simplexes S'_{p+1}, and S'_{n-p-1} of the groups of vertices of S'_n corresponding with the vertices of S_{p+1} and S'_{n-p-1}, these points lie on a same right line through P again, viz.: B and C' on one side and C and B' on the other side of P. If furthermore M and M' are corresponding vertices of S_{p+1} and S'_{p+1} these points lie in parallel normals erected in B and B' on BB' and the line connecting M and M' passes through P. The point of intersection N of BM and $C'M'$ is the vertex of $\overline{S}_{p+1}$ corresponding to the vertex M of S_{p+1}. From CM and $C'M'$ being parallel follows

$$\frac{BN}{MB} = \frac{C'B}{BC} = \frac{C'P - BP}{BP + PC},$$

whilst the relations

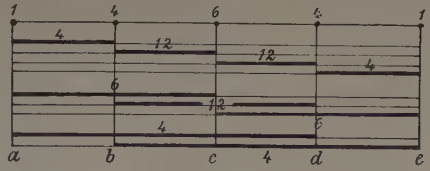
$$\frac{AP}{PA'} = \frac{BP}{PB'} = \frac{CP}{PC'} = \frac{2p+1}{2n-2p-1}$$

and

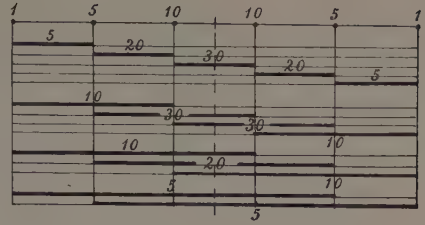
$$\frac{BP}{PC} = \frac{B'P}{PC'} = \frac{n-p-1}{p+1}$$

enable us to express $C'P$ and BP in PC. Substitution gives the result

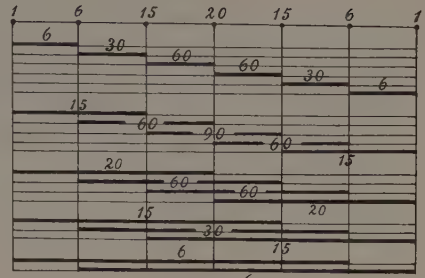
P. H. SCHOUTE. "The section of the measure-polytope M_n of space Sp_n with a central space Sp_{n-1} perpendicular to a diagonal."



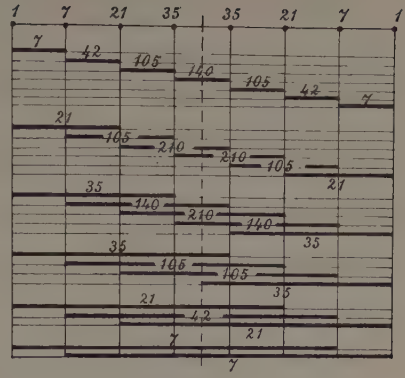
$n = 4$



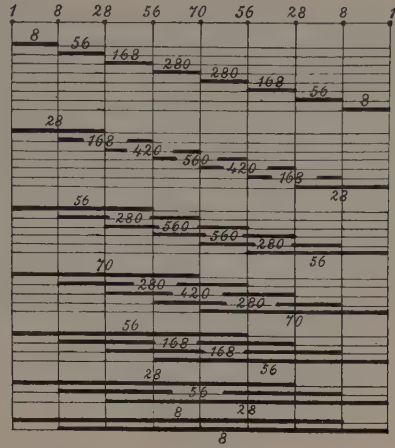
$n = 5$



$n = 6$



$n = 7$



$n = 8$

$$\frac{BN}{MB} = \frac{1}{2p+1}.$$

So the theorem holds:

"If we describe in the spaces Sp_p bearing the bounding simplexes $S_{p+1}\left(\frac{2p+1}{2}\sqrt{2}\right)$ of a regular simplex $S_n\left(\frac{2p+1}{2}\sqrt{2}\right)$ simplexes $S_{p+1}\left(\frac{1}{2}\sqrt{2}\right)$ concentric and oppositely orientated to the original ones we find the $(p+1)\binom{n}{p+1}$ vertices of a $\frac{2p+1}{2n}(M_n)$."

For odd $n = 2n'+1$ we have in particular:

"If we describe in the spaces $Sp_{n'}$, bearing the bounding simplexes $S_{n'+1}\left(\frac{2n'+1}{2}\sqrt{2}\right)$ of a regular simplex $S_{2n'+1}\left(\frac{2n'+1}{2}\sqrt{2}\right)$ simplexes $S_{n'+1}\left(\frac{1}{2}\sqrt{2}\right)$ concentric and oppositely orientated to the original ones we find the $(n'+1)\binom{2n'+1}{n'+1}$ vertices of a $D_{2n'}$."

In connection with the results found above the length $\frac{1}{2}\sqrt{2}$ appearing here for the edges of the new simplexes contains a confirmation.

Mathematics. — "*On five pairs of four-dimensional cells derived from one and the same source.*" By Mrs. A. Boole Stott and Prof. P. H. Schoute.

(Communicated in the meeting of December 28, 1907).

Introduction.

As this paper must be regarded as a short completion of the handbook of the "Mehrdimensionale Geometrie" included in the Sammlung Schubert we keep the notation used there.

We regard in succession each of the six regular cells C_5, C_8, C_{16}, C_{24}, C_{120}, C_{600} of the space Sp_4 and derive from these two new four-dimensional cells. The first, which has the centres K_0 of the edges of the regular cell as vertices is formed by a regular truncation at the vertices as far as the centres of the edges; the second is the reciprocal polar of the first with respect to the spherical space of the points K_0.

Because the regular C_{16} leads us to find the regular C_{24}, the number of pairs of new cells is not six but five.

I. *General observations.*

1. If we understand for the regular cells by e, k, f, r successively the number of the vertices, edges, faces, bounding bodies, by p, q the number of bounding bodies through an edge, through a point, by e', k', f' the number of vertices, edges, faces of the bounding bodies, then besides the relations

$$e + f = k + r \quad , \quad e' + f' = k' + 2$$

of EULER the equations hold

$$qe = re' \quad , \quad pk = rk' \quad , \quad 2f = rf' \ ,$$

out of which number of five we can easily deduce the relation

$$(q - 2)\, e = (p - 2)\, k. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The following table furnishes these quantities for the six regular cells of Sp_4.

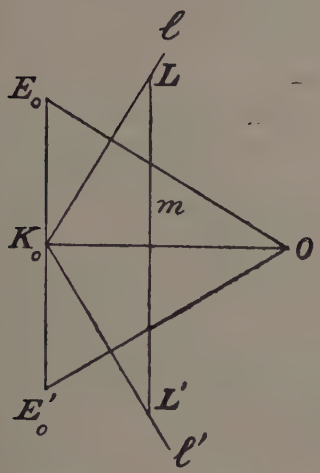
e	k	f	r	p	q	e'	k'	f'
5	10	10	5	3	4	4	6	4
8	24	32	16	4	8	4	6	4
120	720	1200	600	5	20	4	6	4
16	32	24	8	3	4	8	12	6
24	96	96	24	3	6	6	12	8
600	1200	720	120	3	4	20	30	12

2. We shall now endeavour to express the characteristic numbers E, K, F, R of the first of the two new cells — and what is also possible for these P, Q — in the characteristic numbers e, k, f, r, p, q of the regular cell.

"The number of vertices of the new cell is k, i. e. $E = k$."

If we project the regular cell (see the diagram) on the plane through one of the edges E_0E_0' and the centre O, the two new bounding spaces passing through the centre K_0 project themselves according to the perpendiculars l, l' let down out of K_0 on the axes OE_0, OE_0'. The section of the regular cell with a plane normal to the plane of projection in a point lying close to E_0E_0' being an equilateral triangle,

a square or a regular pentagon, with the assumed point always as centre, according to p having the value 3, 4 or 5, the section of the system of the $p+2$ bounding spaces of the new cell passing through K_0 with a space normal to OK_0 — e.g. with a space which according to the normal m to OK_0 is perpendicular to the plane of projection — is a right p-lateral prism, of which the segment LL' of m enclosed between l, l' is the axis and the perpendicular endplanes project themselves in L and L'. From this ensues:



"Through a vertex pass $p+2$ bounding spaces, i.e. $Q=p+2$".

"Through an edge pass three bounding spaces, i.e. $P=3$".

"Through a vertex pass $2p$ edges, so pk is the number of edges, i.e. $K=pk$."

"The system of the bounding spaces consists of two groups, namely of e regular polyhedra with q faces, and r semi-regular polyhedra (e', k', f') with equivalent vertices truncated at the vertices as far as the centres of the edges, i.e. $R=e+r$".

"As the polyhedra of the second group have $e'+f'=k'+2$ faces and a face is common to two bounding spaces, the number of faces is half the sum of qe and $r(k'+2)$ or qe and $pk+2r$, i.e. $2F=qe+pk+2r$".

Thus the result is:

"The first of the two cells, (E, K, F, R, P, Q), deduced out of the regular cell (e, k, f, r, p, q) has the characteristic numbers

$$E=k, \quad K=pk, \quad F=\tfrac{1}{2}(qe+pk)+r, \quad R=e+r,$$
$$P=3, \quad Q=p+2."$$

Here the law of EULER $E+F=K+R$ may serve as verification. In reality the difference of the two members of this equation

$$\begin{aligned} E+F-(K+R) &= k+\tfrac{1}{2}(qe+pk)+r-(pk+e+r) \\ &= k-e+\tfrac{1}{2}(qe-pk) \\ &= \tfrac{1}{2}\{(q-2)e-(p-2)k\} \end{aligned}$$

is equal to zero in consequence of the relation (1).

3. The second of the new cells deduced out of the regular cell is enclosed by the polar spaces of the centres K_0 of the edges with respect to the spherical space through those points, i. e. by the

tangential spaces to that spherical space in those points, i. e. by the spaces in the points K_0 normal to the axes OK_0[1]). By polar inversion of the result found above we arrive with respect to this second new cell at the following results:

"The number of bounding spaces of the new cell is k, i. e. $R' = k$."

"The bounding bodies have $p + 2$ vertices and are double pyramids with a regular polygon with p-sides as base lying in a plane bisecting the connecting line of the vertices at right angles."

"The faces are isosceles triangles."

"In a bounding space lie $2p$ faces, so pk is the number of faces, i. e. $F' = pk$."

"The system of the vertices consists of two groups, namely of e regular vertices and r semi-regular vertices, i. e. $E' = e + r$".

"The number of edges K' is $\frac{1}{2}(qe + pk) + r$."

So the result is:

"The second of the two cells, (E', K', F', R'), deduced out of the regular cell (e, k, f, r, p, q) is bounded by double pyramids with a regular polygon with p-sides as base and has the characteristic numbers

$$E' = e + r, \quad K' = \tfrac{1}{2}(qe + pk) + r, \quad F' = pk, \quad R' = k."$$

It might appear as if it were possible to deduce more pairs of new cells out of the regular cells by doing for the ends F_0 of the axes OF_0 the same as has been done above for the points K_0. This is, however, not the case, because for each regular cell the centres F_0 of the faces form the centres K_0 of the edges of another regular cell which is for the cells C_5, C_{24} dualistically related to themselves a cell of the same kind, for the cells related in pairs to one another (C_8, C_{16}) (C_{120}, C_{600}) a cell dualistically related. And as is immediately evident, the pointgroups E_0 and R_0 can neither lead to new results.

We conclude these general observations with the remark that the two cells deduced from the regular cell (e, k, f, r) show much regularity; of the former the vertices and the edges, of the latter the faces and the bounding bodies are mutually equivalent groups of elements, whilst the faces and the bounding bodies of the former and the vertices and edges of the latter form groups of elements consisting of two subgroups. Do these new cells furnish the maximum amount of regularity for polytopes not entirely regular? We do not intend to go into further details here, as the Mathematical Society at Amsterdam is proposing a prizequestion about what is to be understood by "semi-regular polytopes".

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[1]) The handbook quoted above contains in Vol II pages 256—261 some communications about the corresponding polytopes in the space Sp_n.

The following table shows the results which are obtained by substituting the values of e, k, f, r, p, q for the five different cases. For the sake of completeness those quantities are also included which indicate how many vertices are situated in face and bounding space. We must here notice that, the first cell having two kinds of faces and bounding bodies, we are obliged to take four new quantities, namely the numbers of vertices S_1 and T_1 in face and bounding body of one, the numbers of vertices S_2 and T_2 in face and bounding body of the second kind. Here S_2 and T_2 will relate to the truncating bodies with faces of the same kind and S_1 and T_1 refer to the truncated bodies, where we must then consider as far as S_1 is concerned those faces which the truncated bodies keep in common. We must likewise for the second cell, with two kinds of vertices and edges, introduce the four new quantities P_1', P_2', Q_1', Q_2',

As is evident $T_1 = Q_1' = k'$, whilst $T_2 = Q_2'$ is the number of vertices of the regular polyhedron with q faces.

	e	E, R'	K, F'	F, K'	R, E'	P, S'	Q, T'	S_1, P_1'	S_2, P_2'	T_1, Q_1'	T_2, Q_2'
C_5	5	**10**	**30**	**30**	**10**	3	5	3	3	6	4
C_8	16	**32**	**96**	**88**	**24**	3	5	4	3	12	4
C_{24}	24	**96**	**288**	**240**	**48**	3	5	4	4	12	8
C_{600}	120	**720**	**3600**	**3600**	**720**	3	7	3	3	6	12
C_{120}	600	**1200**	**3600**	**3120**	**720**	3	5	5	3	30	4

In a second part we shall submit each of these five pairs of new cells to a separate investigation.

Mathematics. — "*The analogon of the Cf. of* KUMMER *in seven-dimensional space*". By Dr. J. A. BARRAU. (Communicated by Prof. D. J. KORTEWEG).

(Communicated in the meeting of December 28, 1907).

§ 1. In a preceding communication a method was indicated to generate the *Cff.* in spaces of $(2^\nu - 1)$ dimensions, which can be regarded as analoga of the *Cf.* of KUMMER [1]).

[1]) A quotation in HUDSON's *Kummer's Quartic Surface* (p. 187) drew my attention to the fact, that these *Cff.* have already been obtained by an altogether different method by W. WIRTINGER *(Göttinger Nachrichten*, 1889, page 474; *Monatshefte für Mathematik und Physik* I, page 113; *Mathem. Annalen* 40, page 74). In these papers the varieties are also investigated, for which the elements of such *Cff.* are singular in the same way as those of the *Cf.* (16_6) for KUMMER's quartic surface.

In the following the K^{VII} is under closer investigation especially with a view to the *Cff.* obtainable out of it by omission of certain elements. To this end it is necessary to construct an other diagram than that of eight simplexes, which can only clearly show the *Cff.* (56_{25}); (48_{22}); (40_{19}); (32_{16}); (24_{13}) and (16_{10}), formed by omission of the elements of 1, 2, 3, 4, 5, 6 simplexes, all (except the first) in different types, likewise of *Cff.* (24_9) and (32_{13}), constructed exclusively of fillings (8_3).

§ 2. If we isolate in the *Cf.* of KUMMER a point and a plane not incident to it, the remaining fifteen elements of each kind are divided into a sextuple incident to one of the isolated elements and a nonuple. Each one of the two sextuples forms with the 15 elements of the other kind a free *Cf.* $(6_5, 15_2)$ which means nothing else but that each of the fifteen right lines connecting the *Cf.*-points in one plane bears another *Cf*-plane and reciprocally.

The two nonuples of elements, however, of both kinds together form one *Cf.* (9_4), the structure of which is identical to that of *Cf.* (9_4) III of the classification of MARTINETTI [2]).

This arrangement can be done in $16 \times 10 = 160$ ways.

We can likewise isolate out of K^{VII} in $64 \times 36 = 2304$ ways a point and an Sp_6 not incident to it, by which the sixty-three other elements of each kind are divided into a group of twenty-eight incident to the isolated element of the other kind and a remaining group of thirty-five. The two groups of twenty-eight form together a scheme (28_{12}); each group of twenty-eight with that of thirty-five of another kind a scheme $(28_{15}, 35_{12})$; addition of (28_{12}) and $(28_{15}, 35_{12})$ furnishes a scheme $(28_{27}, 63_{12})$; the two groups of thirty-five form together a scheme (35_{16}).

This arrangement made for the two elements $A1$ is shown in the plate, where the same notations are assigned to the elements as in the diagram of simplexes of which it is a transformation.

We have but to explain how the regular composition indicated by the thicker lines is obtained.

§ 3. Let us first take into consideration the scheme $(28_{27}, 63_{12})$ of points (columns) and Sp_6 (rows). Every number of twelve points on a row lying in two different *Cf.*-Sp_6 lies in an Sp_5; so we can take the *Cf.* to consist of twenty-eight points and sixty-three Sp_5 in Sp_6; each of the sixty-four *Cf.*-Sp_6 of the K^{VII} contains such a *Cf.* (and reciprocally).

[2]) Atti della R. Accademia Peloritana XV.

We find for the complete notation of such a *Cf.* in a way analogous to that of § 6 of the former paper:

	Sp_0	Sp_1	Sp_2	Sp_3	Sp_4	Sp_5
	28	**378**	**2016**	**5040**	**1008**	**63**
incid. to:						
Sp_0	—	2	3	4	6	12
Sp_1	27	—	3	6	15	66
Sp_2	216	16	—	4	20	160
Sp_3	720	80	10	—	15	240
Sp_4	216	40	10	3	—	32
Sp_5	27	11	5	3	2	—

By projection and intersection we find from this in Sp_3 a *Cf.* $(378_{16}, 2016_3)$; a $(2016_{10}, 5040_4)$ and a $(5040_8, 1008_{15})$ of points and right lines, in Sp_3 a $(378_{80}, 5040_6)$ and a $(2016_{10}, 1008_{20})$ of points and planes.

Although the number of *Cf*-points is $28 = 4 \times 7$ one cannot succeed in forming four simplexes S_6 out of the *Cf.*-elements; after isolation of such a simplex (which is possible in several ways) we can form out of the remaining elements (also in several ways) at most a scheme S_5, and then an S_4, S_3, S_2 and S_1 after which of the (28_{12}) an element of each kind is left, mutually not-incident, which we join to an "S_0". In the figure $S_3 + S_2 + S_1 + S_0$ are taken together to a scheme (10_6) which we indicate by T.

The arrangement of the thirty-five remaining elements follows now by our regularly putting down the combinations 3 by 3 of the seven points chosen for S_6; it is evident that the entire diagram contains along the chief diagonal only schemes S or T, whilst outside a couple of new fillings appear amongst which we notice a (10_4), complementary to T.

It is by addition of these partial schemes that we can obtain a great number of *Cff.* included in the total figure; we restrict ourselves to the *forced Cff.* which are those of which each element shows more incidences than are sufficient to determine it and of which for this reason the existence is remarkable from a geometric point of view.

Of the *Cf.* $(28_{27}, 63_{12})$ in Sp_6 the twenty-eight points form evidently with twenty-eight Sp_5 a dual *Cf.* (28_{12}), the same points with the thirty-five other Sp_5 a *Cf.* $(28_{16}, 35_{12})$.

By omission out of (28_{12}) of a S_6 remains a (21_{10}); by omitting S_6 and S_5 a (15_8) remains the scheme of which is *anallagmatic*: each couple of its *Cf.*-Sp_5 has four *Cf.*-points in common. The same number of 15 points forms with 15 other Sp_5 (namely $H1$ as far as $B1$ included out of the number of thirty-five) a *Cf.* (15_7) of which the scheme is complementary to the anallagmatic (15_8).

Out of the *Cf.* (35_{16}) is formed by omission of S_4 a *Cf.* (30_{14}); by omission of T a *Cf.* (25_{12}); of S_4 and T a *Cf.* (20_{10}); of two different T a *Cf.* (15_8), identical to the already mentioned one, its points lie in an Sp_6, the (35_{16}) has in each of the twenty-eight other *Cf.*-R_6 such a (15_8).

If we add to the *Cf.* (30_{14}) a system T out of (28_{12}) a *Cf.* (40_{18}) is formed.

The *Cf.* (35_{16}) is also obtainable out of the diagram of simplexes of the K^{VII}, the simplex A then falls away entirely, of each of the seven other ones three elements of each kind disappear. The diagram (35_{16}) consists of seven systems S_4 in the chief diagonal, mutually connected by fillings (5_2), which all degenerate into (3_2) and (2_2).

By omitting 1, 2, 3, 4 from this S_4 we obtain *Cf.* (30_{14}), (25_{12}), (20_{10}) and (15_8). The (30_{14}) is identical to the already mentioned one, the (15_8) however is of another type: not anallagmatic, neither do its points lie in *one* Sp_6.

§ 5. In each of the Sp_6 formed by intersection of two *Cf.*-Sp_6 of K^{VII} lie 12 *Cf.*-points, of which thirty-two sextuples are also common to a third *Cf.*-Sp_6; such a sextuple lies thus in an Sp_4, the twelve points form with the thirty-two Sp_4 a *Cf.*$(12_{16}, 32_6)$.

We can give to the diagram of such a *Cf.* the following form: (see table p. 507).

If e. g. we take the *Cf*-Sp_5, formed by the intersection of the *Cf*-Sp_6: $A1$ and $A2$, the twelve points become respectively:

$A3 = P1$	$B4 = Q1$
$A4 = P2$	$B3 = Q2$
$A5 = P3$	$C6 = Q3$
$A6 = P4$	$C5 = Q4$
$A7 = P5$	$H8 = Q5$
$A8 = P6$	$H7 = Q6$

The entire *Cf.* consists evidently of two simplexes S_5: P and Q in MÖBIUS-position forming together the part (12_6) whilst moreover every triplet of vertices of one simplex with the three non-conjugate ones of the other lie in *one* Sp_4, i.o.w. each face of one intersects the non-conjugate one of the other.

P							Q					
	2	3	4	5	6		1					
1		3	4	5	6			2				
1	2		4	5	6				3			
1	2	3		5	6					4		
1	2	3	4		6						5	
1	2	3	4	5								6
1								2	3	4	5	6
	2						1		3	4	5	6
		3					1	2		4	5	6
			4				1	2	3		5	6
				5			1	2	3	4		6
					6		1	2	3	4	5	
1	2	3								4	5	6
1	2		4						3		5	6
1	2			5					3	4		6
1	2				6				3	4	5	
1		3	4					2			5	6
1		3		5				2		4		6
1		3			6			2		4	5	
1			4	5				2	3			6
1			4		6			2	3		5	
1				5	6			2	3	4		
	2	3	4				1				5	6
	2	3		5			1			4		6
	2	3			6		1			4	5	
	2		4	5			1		3			6
	2		4		6		1		3		5	
	2			5	6		1		3	4		
		3	4	5			1	2				6
		3	4		6		1	2			5	
		3		5	6		1	2		4		
			4	5	6		1	2	3			

This connection is for the first time possible in Sp_5, the analogon in Sp_3 would be: two tetrahedra in MÖBIUS-position, where each edge of one intersects the non-conjugate one of the other; of this *Cf.* $(8_7, 14_4)$, although it is possible to design the diagram, the execution is evidently impossible.

We find for the complete notation of *Cf.* $(12_{16}, 32_6)$:

	Sp_0	Sp_1	Sp_2	Sp_3	Sp_4
	12	**60**	**160**	**240**	**32**
incid. to					
Sp_0	—	2	3	4	6
Sp_1	10	—	3	6	15
Sp_2	40	8	—	4	20
Sp_3	80	24	6	—	15
Sp_4	16	8	4	2	—

By projecting and intersecting are formed out of these e.g. in Sp_2 a *Cf.* $(60_3, 160_3)$ and a *Cf.* $(160_6, 240_4)$ of points and right lines; in Sp_3 a *Cf.* $(60_{24}, 240_6)$ and a *Cf.* $(160_4, 32_{30})$ of points and planes.

§ 6. The points of Sp_5 can be conjugated one to one to the linear complexes of the usual three-dimensional space, the Sp_4 become linear systems of ∞^4 of these complexes, the *Cf.* $(12_{16}, 32_6)$ can be represented in our space.

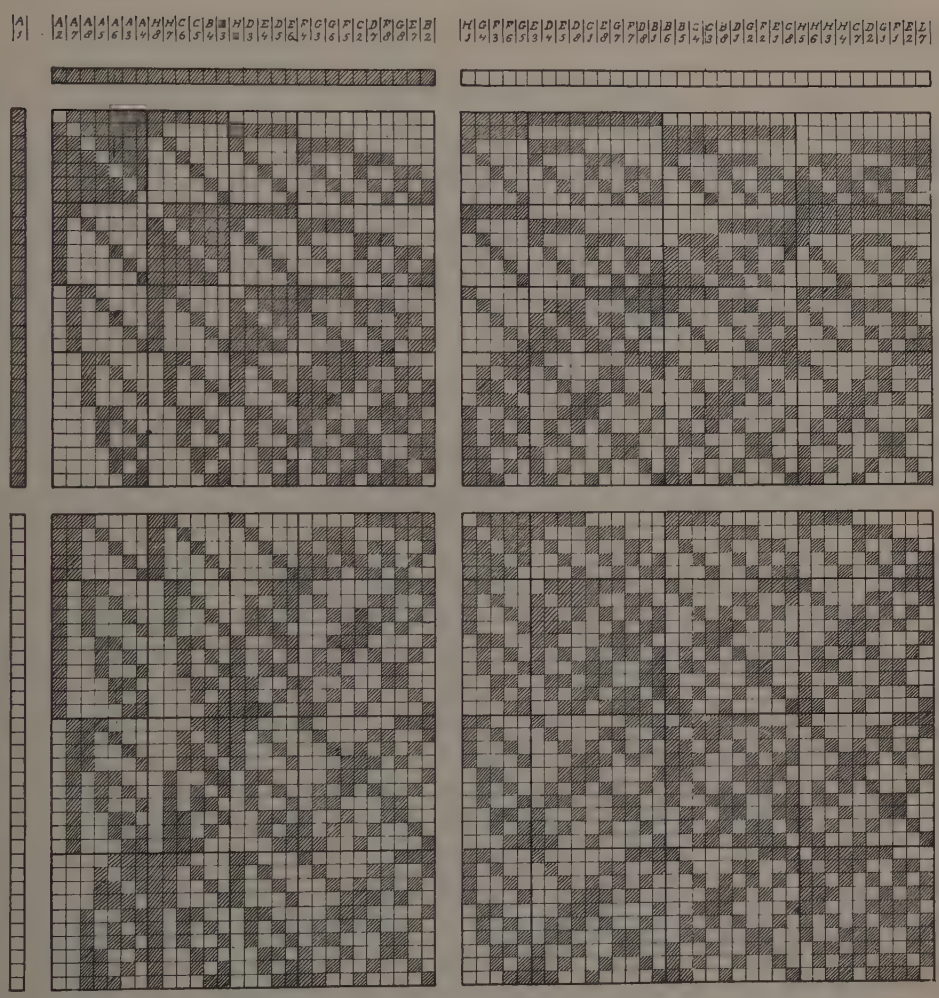
It is however possible to take the twelve complexes simultaneously special and to regard them as right lines, the thirty-two Sp_4 then become linear complexes which each contain a sextuple of the right lines; the *Cf.* $(12_{16}, 32_6)$ is then realized in right lines and linear complexes of our space.

We can easily give line-coordinates for such a number of twelve right lines by omitting from the point-coordinates of the twelve points a couple, e. g. X_7 and X_8, and by letting the six remaining ones satisfy the fundamental relation

$$X_1 X_6 + X_2 X_5 + X_3 X_4 = 0$$

So we obtain e. g. the right lines

J. A. BARRAU. "The analogon of the *Cf.* of Kummer in sevendimensional space."



$P1 = (c,-d,-a,\ 0,-g,\ h)$ $\quad$ $Q1 = (d,\ c,\ 0,-a,\ -h,-g)$

$P2 = (d,\ c,\ 0,-a,\ h,\ g)$ $\quad$ $Q2 = (c,-d,-a,\ 0,\ g,-h)$

$P3 = (0,-f,\ g,-h,-a,\ 0)$ $\quad$ $Q3 = (f,\ 0,\ h,\ g,\ 0,-a)$

$P4 = (f,\ 0,-h,-g,\ 0,-a)$ $\quad$ $Q4 = (0,-f,-g,\ h,-a,\ 0)$

$P5 = (g,-h,\ 0,\ f,\ c,-d)$ $\quad$ $Q4 = (h,\ g,-f,\ 0,\ d,\ c)$

$P6 = (h,\ g,\ f,\ 0,-d,-c)$ $\quad$ $Q6 = (g,-h,\ 0,-f,-c,\ d)$,

if besides is satisfied

$$ch + dg = af - gh = 0.$$

The peculiarity appearing with this example taken for simplicity's sake, that the right lines show mutually some incidences, is lost by submitting the coordinates in Sp_5 first to a linear transformation.

In the same way, indeed, we can formulate for all *Cff.* indicated in spaces of a lower number of dimensions an analytical definition by deducing the coordinates of their elements from those of the elements of K^{VII}.

Chemistry. — "*On the constitution of* VAN GEUNS's *oxymethyldinitrobenzonitrile*". By Dr. J. J. BLANKSMA. (Communicated by Prof. A. F. HOLLEMAN).

By the action of potassiumcyanide on meta-dinitrobenzene in methylalcoholic or ethylalcoholic solution, LOBRY DE BRUYN [1]) obtained in 1882 the oxymethyl- or oxyethylnitrobenzonitrile $C_6H_3(OCH_3)CN\,NO_2$ 1. 2. 3.

The investigation of these substances was continued afterwards by VAN GEUNS [2]) who succeeded in saponifying these nitriles to acidamines and in preparing the corresponding acids thereof. At the same time VAN GEUNS showed that in both substances a further nitro-group can be introduced by nitration with nitric and sulphuric acids thus yielding the compounds $C_6H_2(OCH_3)CN(NO_2)_2$ m.p. 113° and $C_6H_2(OC_2H_5)CN(NO_2)_2$ m.p. 63°, These two compounds contain a movable nitro-group which may be readily replaced by OH, OCH_3, NH_2, $NHCH_3$, NHC_6H_5 etc.

As, however, the place where the nitro-group had been introduced had remained unknown, the constitution of these derivatives was consequently also unknown.

When VAN GEUNS, owing to his departure for India was obliged

[1]) Recueil **2**, 205.

[2]) Dissertation Amsterdam 1903.

to discontinue this research I tried at the request of the late Prof. LOBRY DE BRUYN to determine this constitution. After a few trials which led to no result the method was followed which had proved successful in the determination of the constitution of 2.3.4 trinitrophenetol[1]). The constitution of that substance was shown to be:

$$C_6H_2(OC_2H_5)(NO_2)_3\ 2.3.4 \rightarrow C_6H_2(OC_2H_5)NH_2(NO_2)_2\ 1.3.2.4 \rightarrow C_6H_3(OC_2H_5)(NO_2)_2\ 2.4.$$

Oxymethyldinitrobenzonitrile was now treated in an analogous manner; by the action of alcoholic ammonia one NO_2-group was replaced by NH_2 and this was then in turn removed by diazotation and boiling with alcohol. In this manner was obtained an oxymethylnitrobenzonitrile (m.p. 126°) $C_6H_2(OCH_3)\ CN(NO_2)_2 \rightarrow C_6H_2(OCH_3)\ CN.NH_2NO_2 \rightarrow C_6H_3(OCH_3)CN.NO_2$.

This shows that the NO_2-group at 3 is replaced by NH_2 as otherwise the original oxymethylnitrobenzonitrile $C_6H_3(OCH_3)CN.NO_2$ 1.2.3 m.p. 171° would have been reobtained. Now it remained only to determine the constitution of this substance. On treatment with nitric and sulphuric acids an oxymethyldinitrobenzonitrile was obtained which melts at 71° and which possesses the following constitution: $C_6H_2(OCH_3)CN(NO_2)_2$ 1.2.4.6 [2]).

The constitution of this substance was determined in the following manner. If this compound is treated in alcoholic solution with ammonia or methylamine the OCH_3 group is readily substituted by NH_2 or $NHCH_3$ and dinitrocyano-aminobenzene m.p. 219° or dinitrocyano-methylaminobenzene m.p. 161° is formed which substances were prepared previously from the corresponding oxyethyl compound [3]).

The oxymethylnitrobenzonitrile m.p. 126° was then heated at 150° with hydrochloric acid for 5 hours. On opening the tube a gas escaped which burnt with a green-bordered flame (CH_3Cl) whilst in the tube there were present crystals which after recrystallisation from water melted at 228° and proved to be 5-nitrosalicylic acid (C_6H_3 COOH, OH, NO_2 1. 2. 5.) In the motherliquor the presence of NH_3 was detected, formed by saponification of the cyano-group. For the purpose of identifying the substance obtained a little of the preparation was mixed with an equal quantity of 5-nitrosalicylic acid (m.p. 228° prepared by nitration of salicylic acid [4]). The melting point

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[1]) Recueil **27**, 49.

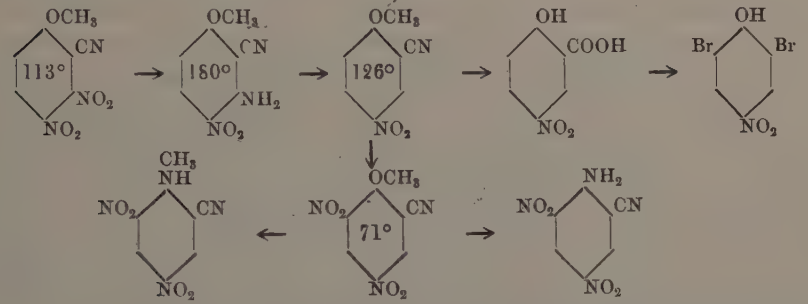
[2]) This shows that in oxymethylnitrobenzonitrile m.p. 126° the nitro-group is placed on 4 or 6.

[3]) BLANKSMA. Rec. **20**, 413. **21**, 274.

[4]) HÜBNER. Ann. **195**, 31.

was not altered thereby. Both preparations could also be converted readily into 2-6-dibromo-4-nitrophenol m.p. 141° by treatment with bromine water [1]).

The following reactions were applied:

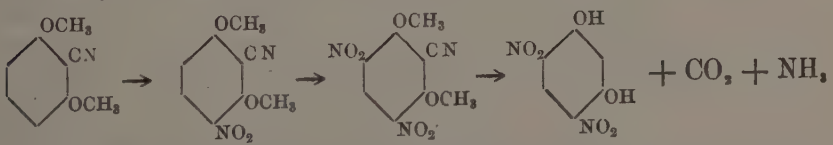


This proved that the constitution of the oxymethyldinitrobenzonitrile prepared by Van Geuns is $C_6H_2(OCH_3), CN, (NO_2)_2$. 1, 2, 3, 4.

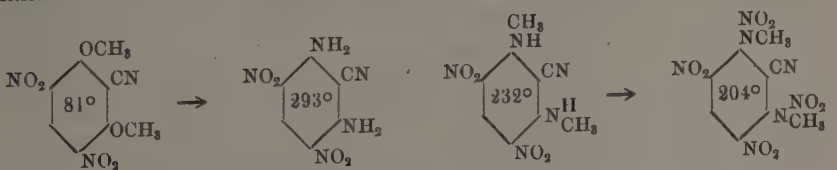
At the same time it was shown that the movable NO_2-group in this substance is placed at 3; consequently we now know the constitution of the compounds obtained from it by substitution of the NO_2-group by OH, OCH_3 etc.

Finally, the constitution was determined of the dinitrodimethoxybenzonitrile obtained by the nitration of $C_6H_3(OCH_3)_2 CN$ 1, 3, 2, or of the nitrodimethoxybenzonitrile $C_6H_2(OCH_3)_2 CN.NO_2$ 1, 3, 2, 4. [2]).

This compound was converted into 4-6-dinitro-resorcine m.p. 215° by being heated for 5 hours at 150° — 160° with hydrochloric acid (30 % HCl); from this follows that its constitution is $C_6H(OCH_3)_2 CN (NO_2)_2$ 1. 3. 2. 4. 6.



4.6 dinitro-2-cyano 1.3 dimethoxybenzene on treatment with alcoholic ammonia or methylamine readily yields compounds which perfectly resemble the compounds which have been obtained in a similar manner from 2.4.6 trinitroresorcinoldimethylether.



[1]) Lellmann and Grothmann. Ber. 17, 2731.

[2]) Dissertation Van Geuns. p. 69.

Geophysics. — *"Registration of earth-currents at Batavia for the investigation of the connection between earth-current and force of earth-magnetism."* By Dr. W. VAN BEMMELEN at Batavia. (Communicated by Mr. J. P. VAN DER STOK).

(Communicated in the meeting of December 28, 1907.)

Notwithstanding the great progress in our knowledge of the phenomena of earth-magnetism, the desired improvement has not yet been noticed in the explanation of these phenomena.

That the different variations to which the magnetic needle is liable are the consequence of the changes of electric currents has become highly probable and the place, too, where in that case the currents are to be found is no longer entirely unknown to us.

So SCHUSTER [1]) has proved that the daily variation is in general caused by extra-terrestrial currents, whilst I myself have indicated [2]) that this is likewise the case for that part of the magnetic disturbances which shows a regular daily variation.

Suchlike electric currents have; however, not been shown experimentally and their indication in those unapproachable regions is for the present not to be expected.

Only *one* part of the earth is accessible to us, viz. the outer crust and numerous are the investigations on the electric currents circuiting in that crust.

However, all these investigations have but poorly advanced our knowledge about the connection between those currents and the magnetic variations.

The reason is not only to be found in the great experimental obstacles and the lack of cooperation in the various investigations, but especially in the complicated relations of the system of currents in those zones of the earth where those investigations have been made, viz. between 40° and 70° latitude.

My supposition that in the equatorial zone, just as for other geophysical phenomena, simpler relations must exist, has been proved to be right by the investigations which I wish to communicate here.

The annotations of the earth-current executed by me these last three years at the observatory of Batavia can be divided into two series.

During the first period, March—November 1905, I registered the

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[1]) Phil. Trans. Vol. 180, p. 667.

[2]) Natuurk. Tijdschrift voor N. I. Dl. 63, p. 227.

earth-current between Cheribon and Batavia, with the aid of the intercommunal telephone-line. The variations were written down photographically, the velocity of the registering strip amounted to the usual 15 mm. an hour.

The important results obtained by this method incited me to go on and in the period now come to a close I could at night make use, by the kind cooperation of the officials of the Telegraph Service, of different telegraph-lines (to Anjer, Buitenzorg, Soekaboemi, Billiton, Poerwakarta, Cheribon, Samarang, Soerabaja and Makassar); greater velocity of registration was applied too (60 and 240 mm. an hour).

Besides continuing the different registrations of the earth-current during longer or shorter time to obtain statistic results, I also made experiments. When the registration pointed to a new connection between earth-current and magnetic variation, other registrations of the earth-current were organised to get a closer investigation of that connection. When questions on the influence of wire or groundplate cropped up, it was tried to answer them by experimentation.

The instrumental arrangement was contrived in such a way, that on a strip 20 cm. wide beside the variations of two earth-current circuits those of the corresponding magnetic components were noted down. Corresponding means here: the component normal to the direction of the earth-current circuit.

The sensibility was chosen in such a way, that the corresponding variations of earth-current and magnetic force did not differ too much in magnitude. To this end great sensibility of the magnetic variation instruments (up to 0.1 γ an mm.) was necessary; as however only the relative position during one night was considered it was easy to arrange those variation instruments quite simply.

I intend to give a more extensive communication about this at some other time.

The daily variation.

The obstacles met with in investigating the daily variation were very great. For the most important part of that variation takes place about noon, but during the day time the electric field of Batavia is disturbed by the electric tram and moreover I was allowed the use of the lines only at night.

If still I succeeded in coming to useful results, this is due to the kindness of the Superintendent of the Intercommunal Telephone Company, Mr. S. W. Baints, who allowed hourly readings of the amount of the earth-current to be done at the office of the Company at Batavia. The hours were 8^h45^m A. M., 9^h45^m A. M. etc. until

4^h45^m P. M.; an ordinary galvanometer with pointer was used.

I have chosen from these readings those falling on magnetically very calm days and evincing moreover not to have suffered from disturbances on the line or from other irregularities. For those same days I have used the observatory-notation during the hours of the night (6 P. M.—5 A. M.).

Two unknown quantities remained, viz. the ratio of the values of the scale division and the difference of the central position.

The former I determined one evening during a magnetic storm at the office of the Company by alternately reading the galvanometer and allowing the Observatory to register. The reduction to a same central position I got to a plausible result by using the Sunday notations. For, on those days I could use the line already after 12 at noon and from a score of magnetically fairly calm Sundays I deduced the difference between the hours $4^3/_4$ and 6 p.m.

Graphically I then interpolated the 24 values of the hours of the day.

For the employed magnetically calm days finally was calculated the daily variation of the magnetic component normal to the direction Cheribon-Batavia from the Buitenzorg magnetograms.

Daily variation of the earth-current Cheribon-Batavia and of the magnetic horizontal component normal to that direction.

Earth-current in Volt per Kilometer $\times 10^{-5}$ (Direction Ch.-Bat. = +)	Magnetic Component in 10^{-5} C. G. S. (NE = +)	Earth-current in Volt per Kilometer $\times 10^{-5}$ (Direction Ch.-Bat. = +)	Magnetic Component in 10^{-5} C. G. S. (NE = +)
1 a. m. — 38	—11.1	1 p. m. + 33	22.2
2 — 36	— 9.6	2 — 6	13.9
3 — 40	— 9.3	3 — 33	3.3
4 — 33	— 7.9	4 — 40	— 4.5
5 — 32	— 6.5	5 — 32	— 9.4
6 — 22	— 3.0	6 — 23	—10.2
7 11	3.4	7 — 51	—11.0
8 81	8.8	8 — 60	—13.1
9 153	15.9	9 — 51	—13.6
10 154	24.2	10 — 47	—14.6
11 117	29.5	11 — 46	—13.7
12 84	28.5	12 — 39	—12.9

Out of the curves, indicating according to the above numbers the daily vibration of earth-current and magnetic component, is evident:

that this vibration for them corresponds;

that the direction of the earth-current is such that it can be regarded as causing the variations of the magnetic component;

further: *that the magnetic component is retarded with respect to the earth-current* and finally;

that the ratio of the amplitudes of corresponding vibrations decreases with the duration of that vibration, so that those of the earth-current are relatively larger with a shorter duration.

The chief maximum in the afternoon is reached by the earth-current about an hour and a half earlier, the chief maximum at night about two hours earlier.

The secondary vibration in the evening-hours is for the earth-current much stronger.

It is an indication to apply here the harmonic analysis and to employ for it the formula

$$A = A_n \sin n\,(t + c_n).$$

The results of the harmonic analysis confirm in full what the mere observation taught us.

Especially the increase of the earth-current as the duration of the corresponding variation of the earth-magnetism becomes shorter is very distinctly expressed.

This dependence can be pretty accurately expressed by the following formula.

Let the amplitude of the magnetic component be M; the duration expressed in days T, and the amplitude of the earth-current A, then

$$A = 0.8 \sqrt[4]{\frac{1}{TM}}.$$

The values in the above column "calc" (on the next page) have been computed with the aid of this formula.

Yet not much value must be attached to that correspondence, as the higher terms of the harmonic analysis are very untrustworthy on account of the inaccuracy of the hourvalues employed.

The difference of phase increases regularly as far as the 5th term, and then drops again to the value it had for the 3rd term; but the phase differences found for the higher terms deserve little confidence.

I have been successful in obtaining a confirmation of a part of these results with the aid of the cable Batavia—Billiton. The four months March—June 1906 gave for the nightly course proper results.

	AMPLITUDE				PHASE.		
	Earth-current Volt. p. K.M. $\times 10^{-5}$	Magn. Comp. C.G.S. $\times 10^{-5}$	Earth-current / Magn. Comp. Observ.	Earth-current / Magn. Comp. Calc.	Δ C—O		Earth-current	Magn. Comp.	Earth-current / Magn. Comp.
A_1	75	18.6	4.0	3.9	−0.1	C_1	299°	286°	13°
A_2	44	7.6	5.7	5.9	0.2	C_2	73	53	20
A_3	28	3.0	9.4	8.0	−1.4	C_3	122	98	24
A_4	9	0.9	10.0	10.8	0.8	C_4	53	15	38
A_5	7	0.5	13.1	13.6	0.5	C_5	103	56	47
A_6	6	0.4	15.7	15.9	0.2	C_6	57	34	23

The variation of the corresponding magnetic component changed its nature pretty much (as was to be expected) during these months. The earth-current really followed this variation whilst the maxima and the minima kept preceding those of the magnetic component. Out of the average for the 4 months this is obvious.

	7	8	9	10	11	Midnight	1	2	3	4	5
Earth-current Volt. p. KM. $\times 10^{-5}$	0	−27	−25	−29	−17	12	20	28	26	0	−14

Magnet. comp. 0 —2.1 —3.7 —5.3 —5.5 —4.9 —4.1 —3.3 —3.4 —4.5 —5.4 10^{-5} C. G. S.

For the earth-current the maximum comes one hour, the minimum about half an hour earlier. Let us suppose this minimum to belong to the preceding vibration of $3^h\ 30^m$ duration, then the difference in phase is 26°.

The ratio of the amplitude is 26°.0, whilst I deduced roughly out of the Sunday notation for the great vibration 16.0 when the cable was at my disposal from 0^h till 4^h p.m.

So we meet here too with decrease of the ratio between earth-current and corresponding magnetic component together with increase of the duration of the vibration, but by the side of it a much stronger earth-current than for the line Cheribon—Batavia.

Annual inequality in the daily vibration.

At Batavia the amplitude of the daily vibration of the magnetic force is liable to a single- and a double-yearly inequality, where the maxima are attained in March and September, the minima in January and in June. The two maxima and the two minima are of the same magnitude.

From the continued measurements at the office of the Intercommunal Telephone Company I could deduce that the variations of the earth-current show the same annual inequality.

This series of measurements shows two breaks.

First in January '06 the lines were permanently disturbed and secondly in August '06 errors seem to have slipped into the observations, on account of which repeatedly improbably large values were read. After my having pointed this out, the readings in December next were again serviceable.

Out of each month I have taken those days which were in the first place magnetically calm and for which in the second place as much as possible complete and useful readings of earth-currents were at hand.

Of the mean hourvalues for each month was then taken the difference of the smallest and the greatest value.

The maximum generally fell in with the $8^3/_4$ a.m. or $9^3/_4$ a.m. observation, the minimum with that of $3^3/_4$ p.m. or $4^3/_4$ p.m.

These differences expressed in Volt pro kilometer $\times 10^{-6}$ follow here.

	J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.
1905		**266**	194	208	190	129	**127**	**127**	170	**173**	167	131
1906		171	**177**	127	125	**109**	135	233 (?)				122
1907	**81**	**118**	113	90	88.							

Notwithstanding the imperfection in these measurements, the double annual period and its correspondence with that of the magnetic component is so distinctly expressed that doubt is not possible.

Variations of short duration.

The second period of registration, November 1905—October 1907, was chiefly devoted to the study of the connection of the vibrations of short duration in earth-current and magnetic component.

The usual velocity of registration was here 1 mm. a minute, which with sharp photographic lines allows the measurement of variations with the duration of half a vibration of 0.2 to 0.3 minute, but in numerous nights the velocity was enlarged to 4 mm. a minute, when it was possible to measure accurately differences of time of 0.1 minute.

By the continued registration of the earth-current along different lines, each one accompanied by that of the corresponding magnetic component, an extensive material of curves was collected, from which in general the following could be gathered.

For the earth-current along about east-west lines [1]) to each vibration answers a similar one of the magnetic component. For that of the nearly north-south lines that correspondence seems also to exist in part, but it is greatly disturbed by the circumstance that the earth-current keeps following more or less the vibrations of the east-west line.

So also near the equator we find complicated phenomena, but only in part, for as far as the east-west current is concerned we meet with such a striking correspondence that it is possible to deduce simple laws; the two same laws, which were found for the daily variation:

1. *the vibration in the earth-current precedes that of the magnetic component with a certain difference in phase;*

2. *the ratio of amplitude of earth-current and magnetic component decreases when the duration of the vibration increases;*

---

[1]) The east-west lines were:

		Direction	Distance
Bat.	— Anjer	W 6° N	106 K.m.
„	— Poerwakarta	E 40 S	78 „
„	— Cheribon	E 18 S	200 „
„	— Semarang	E 12 S	406 „
„	— Soerabaja	E 10 S	665 „
„	— Makassar	E 5 N	1486 „

The north-south lines were:

		Direction	Distance
Bat.	— Billiton	N 13° E	392 K.m.
„	— Buitenzorg	S 5 E	46 „
„	— Soekaboemi	S 9 E	84 „

appear distinctly from this material of registering curves with its thousands and thousands of shorter and slower vibrations.

To deduce the real amplitudes and phases of those variations we should have to execute for each separate case an immensely extensive harmonic analysis and, this being quite impossible, corresponding variations had to be chosen and measured discriminately. Therefore all the measurements have been done by me personally.

The precedence of the earth-current.

This precedence was rule; it was quite exceptional if it was not met with. When choosing cases for measurement I always avoided those where by a superposed oscillation of greater length and amplitude the time of the turning point was made to appear much earlier or later.

The difference of phase proved to vary from case to case but to be already constant in the mean of a small number.

For the lines Batavia—Soerabaja and Batavia—Poerwakarta the difference in phase was determined with respect to that of Batavia—Cheribon and not with respect to the magnetic component.

For the lines Batavia—Buitenzorg and Batavia—Billiton the deduction was accompanied by great difficulties, as perfectly corresponding cases between earth-current and the magnetic component seldom made their appearance on account of the interference of the east-west component.

For Batavia—Soekaboemi I have therefore desisted from making a calculation and the difference in phase for the two first lines must be mentioned with reserve.

Difference in phase.

Batavia—Poerwakarta	22°	Batavia—Buitenzorg	23° (?)
„ —Anjer	14	„ —Billiton	28 (?)
„ —Cheribon	22		
„ —Semarang	36		
„ —Soerabaja	32		

The difference in phase found here for Cheribon and Billiton shows a striking resemblance to that found for the daily variation.

Cheribon.	
Variations of short duration.	Average of 6 terms of the daily variation.
22°	27°

Billiton

Variations of short duration.	Nightly variation.
28°	26° (?)

Ratio of amplitude.

The accurate indication of the moment of maximum or minimum of a vibration is sooner impossible than the measurement of an amplitude on account of the interference of smaller superposed variations.

I have therefore been able to select a much greater number — 346 — of cases for measurement; the results are as follows:

Batavia—Cheribon.

Amplitude Earth-current in Volt p. Kilom.
Amplitude magnetic component in C.G.S.

Duration of half vibration.	Magnetic Component in 10^{-5} C. G. S.							
	0.23	0.37—0.40	0.50—0.79	0.90—1.50	1.7—1.8	2.8—3.2	5.5—7 6	15.1—18.6
0.3 m.	23 3	22.1	22.4					
0.5	24.3	23.8	21.4	22.8				
0.8		23.6	22.3	19.4				
1.2		22.4	21.1		22.5			
3.7			19.2	21.1		19.1		
7.6			18.3		16.9	17.8	15.6	
15.2				14.8		14.3	14.0	
29.0						15.9		
36.3							12.1	
39.6								10.8
120.—		15.7						
144.—			13.1					
180.—				10.0				
240.—						9.4		
360.—							5.7	
720.—								4.0

According to this table the increase of the amplitude when the duration of the vibration diminishes seems to reach a maximum value at 0.5 min. and moreover the ratio of amplitudes seems to be dependent on the amplitude itself and in such a way that with equal duration it increases with the diminishing of the amplitude.

A complete confirmation of these results was found in 312 cases for the Anjer-line.

Batavia-Anjer.

$$\frac{\text{Amplitude Earth-current in Volt p. K.m.}}{\text{Amplitude Magnetic Component in C. G. S.}}$$

Duration of half vibration.	Magnetic Component in 10^{-5} C. G. S.				
	0.19—3.0	0.53—0.68	1.36—1.83	2.30—3.89	5.20—5.48
0.4 min.	96	95	97		
0.6	97	104	95		
0.9	86	90	89		
1.8	92	84			
2.3			64		
5.3		86	74		
6.3				68	
7.7	83				
9.2			68		
9.7					60
10.8		68			
12.3			73		
13.6		58			
19.9				75	
22.0					63

Here too with a short duration a maximum is found, viz. with 0.6 min.

Beside this correspondence we find the unexpected result, *that the Batavio—Anjer current is about four times stronger than the Batavia —Cheribon current.*

That ratio is constant for different duration of vibration, as was proved by the arrangement according to groups of mean equal duration.

Duration of half a vibration	Batavia-Anjer / Batavia-Cheribon
0.33 min.	4.28
0.54	3.94
0.66	4.36
0.76	4.21
0.96	4.21
1.67	3.79
5.6	4.25
7.6	3.84
15.1	5.21
Mean	4.23

When we wish to compare the values for the ratio of amplitudes of magnetic force and earth-current of vibrations of shorter duration with those of the six terms of the daily variation, it is best to inscribe all values in one diagram with abscis $\sqrt[4]{M}$ and ordinate $\sqrt[4]{1/T}$. When now the formula $A = 0.8 \sqrt[4]{\frac{1}{TM}}$ found above for the 6 terms of the daily variation will still hold, then the values which lie on the same radius-vector through the origin must be the same.

It is evident that this is only the case for the middle part of the diagram, namely for the radius-vector where $0.8 \sqrt[4]{\frac{1}{TM}} = 150$.

The ratio $\sqrt[4]{1/T}$ and $\sqrt[4]{M}$ is therefore $= 2$.

If the amplitude of the magnetic component is relatively larger, then the radii with equal values are bent gradually to the axis of abscissae, and if $\frac{1}{T}$ is relatively larger, then they are bent to the axis of ordinates.

For a great $\frac{1}{T}$; i. e. for a duration of vibration of about one minute, they turn again to the axis of ordinates and a maximum seems to be formed.

It will be possible to force these curves in a formula, but we

must not expect that that formula will give the real formula, as it must be very complicated on account of the nature of the phenomenon.

What according to me is clear from the diagram is that the ratio of amplitudes for the vibrations of shorter duration will gradually pass into those of longer duration (the six terms of the daily variation) and so that between these two phenomena there is also a gradual transition.

The results for the earth-current Buitenzorg-Batavia and Billiton-Batavia are again uncertain on account of the lack of agreement with the magnetic component. I found:

Buitenzorg-Batavia.

Duration of half a vibration	Amplitude Magn. Comp.	Amplit. earth-current in V. p Km. / Amplit. magn. Comp. in C. G. S.	Number of cases
1.1 min	0.38 J.	75	20
3.7	0.96	79	20
7.6	1.20	71	28
			68

Billiton—Batavia.

0.7	0.39	63	14
1.3	0.29	74	14
3.5	0.49	66	14
7.0	0.50	58	23
22.4	1.44	41	5
			70

If indeed these figures are trustworthy then the ratio of amplitudes of the earth-current with respect to the magnetic component decreases here too when the duration increases.

The increase at the outset with very small duration is also found in the above figures, even in much greater ratio than for the Anjer- and Cheribon-current.

The numbers for Buitenzorg are a little smaller and for Billiton smaller than for Anjer, but still larger than for the Cheribon-current.

The line to Poerwakarta I had but for two nights at my disposal.

I have then allowed the Poerwakarta-current to be registered at the same time as the Cheribon-current and I have found a complete correspondence between them for vibrations of a duration from 0.8 to 15.5 minutes.

Not before the last months of the registering-period have I extended the investigations to the lines to Semarang and Soerabaja and to my surprise I found a pretty great deviation from the circumstances which appear on the lines Cheribon and Anjer.

Semarang—Anjer.

Duration of half a vibration	Amplitude Anjer-current 10^{-6} V. p Km.	Amplitude Anjer / Amplitude Semarang	Number of cases
0.33 min.	38	1.70	20
0.69	74	2.06	20
1.04	61	2.16	20
1.71	57	2.32	20
7.43	160	3.96	20
20.81	290	4.61	16
			116

Soerabaja—Anjer.

0.37	43	2.17	20
0.93	92	2.87	20
2.65	110	3.27	20
11.85	326	4.71	20
34.28	739	7.16	7
			87

So the influence of the duration on the amplitude of the earth-current is here much greater for the Semarang- and Soerabaja-current than for the Anjer-current.

It is remarkable that here too the increase of the influence with the duration takes place about according to $\sqrt{}^{1}/_{T}$.

With respect to the first value for $t = 0.33$ min., respect. 0.37 minutes duration we get:

	$\sqrt[4]{\frac{t_n}{t_0}}$	$\sqrt[4]{\frac{A_x}{A_0}}$		$\sqrt[4]{\frac{t_n}{t_0}}$	$\sqrt{\frac{A_x}{A_0}}$
Semarang	1.20	1.21	Soerabaja	1.26	1.32
	1.33	1.27		1.64	1.51
	1.51	1.36		2.38	2.17
	2.18	2.33		3.10	3.30
	2.82	2.71			

Direct comparison of the Batavia-Semarang current to the magnetic component furnished: (June 18—21, '07):

Duration of half a vibration	Amplit. of the Magnet. Comp. in 10×10^{-5} C.G.S.	Amplit. earth-current in Volt p. K.M. / Amplit. earth-current in C.G.S.	Number of cases
0.6 min.	1.3	43	14
1.0	0.9	35	15
1.4	0.9	31	13
5.0	0.7	18	10
11.5	2.2	13	9

The last two values fit in very well with the scheme of the values found for the Cheribon-current, but the first three show a much quicker increase when the duration of the vibration is shorter.

This peculiar increase immediately strikes one when regarding the registered lines. In order to investigate whether that increase of the influence of the duration was connected with the increase of the distance of the two stations between which the earth-current was measured, I asked for and obtained direct connection with Makassar. The loss by defective isolation on the line however was so great, that the real distance had not obtained any lengthening of importance.

On the trustworthiness of the results.

A certain doubt has always been left when observing earth-currents whether the results arrived at do give an idea of the real existing earth-current.

According to SCHUSTER (Terr. Magn. III, p. 130) the intensity of the current is really to be determined by switching on in the circuit a cell of known E. M. F. I have therefore always used this

nection with Cheribon I immediately closed the new one, in consequence of which the depolarisation-current passed through the second galvanometer.

I actually found the polarisation with its characteristic qualities, but its intensity was hardly more than a few percents of the chief current and thus really too small to serve as cause of difference in phase and change in ratio of amplitudes.

After this investigation I have placed a set of non-polarising ground-plates (amalgamated zinc plates immersed in a solution of $Zn\,SO_4$ in porous pots) [1]) on the garden of the Observatory, and repaired to Cheribon to place a corresponding set there. The repetition of the experiment described above showed really the non-appearance of polarisation.

After this I connected one of the two telephonewires between Batavia and Cheribon with the old polarising ground-plates, the other with the new non-polarising ones, and allowed the two earth-currents to register simultaneously on the same strip with the same sensibility and a velocity of registration of 24 cm. an hour together with the magnetic component. The experiment could hardly have been taken more accurately.

As I expected the result for the difference in phase was a very slight influence in the sense mentioned above; for the ratio of amplitude I found for *one* night also a very small influence in the expected direction, but during two other nights a somewhat greater difference in opposite sense. I think I must attribute those last influences to the unavoidable inaccuracy of the determination of the values of the scale division (by switching on a cell of known E. M. F.).

At any rate I had proved sufficiently that the current of polarisation was not the cause of the found phenomena, so I can take those phenomena to be real.

Connection between earth-current and magnetic force.

If we wish to investigate more closely the connection between the variation of earth-current and magnetic component it is necessary to regard the variations of the latter quite by themselves.

The general rule holds at Batavia that the two horizontal components change simultaneously, i. e. that generally between the turning-points of X and Y only a small difference in time exists and that on the other hand Z generally has a difference of phase of 90° with X and Y.

Ad. Schmidt (Met. Zeitschrift 1899) has pointed out, that the

[1]) C. A. Brander. Inaugural Dissertation, Helsingfors, 1888.

variations of the magnetic component might be explained by the passage of electric current vortices.

Following this explanation we should have to conclude for Batavia to the passing of vortices whose centre remains far from Batavia, so that only the outside slightly bent pieces of current pass by.

So as a first approximation we may assume that the extra-terrestrial current is almost rectilinear at Batavia and when varying in intensity has but slight oscillations in direction.

The average direction must be $WSW - ENE$ for almost without exception an increase of the N-component is accompanied by a weakening of the E-component and so $\Delta X > \Delta Y$.

That current we really find back in the diagram of the equipotential lines of the daily variation according to SCHUSTER-VON BEZOLD, [1]) which equipotential lines follow at first approximation the current-lines.

Also for the explanation of the phenomenon found by me of the *earthmagnetic after-disturbance*, it was a matter of fact to take a current encircling the earth and this current too had to have a suchlike direction as was mentioned above, but the angle with the equator was at Batavia much smaller than is found now.

Each varying extra-terrestrial current will induct an intra-terrestrial one and the magnetic variation observed at the surface is the sum of the influence of the two. LAMB (see the paper quoted above of SCHUSTER on the daily variation) proves that the ratio of the potential of the primary and the secondary field is complex and that therefore difference in phase exists. The horizontal component caused by the extra-terrestrial current is in advance compared to the one generated by the inducted currents; so the resulting component will be in retardation compared to the extra terrestrial current.

By SCHUSTER however no difference of phase is found for the vertical force and LAMB has pointed to the fact that this can be the consequence of increasing electric conductivity of the earthstrata towards the depth. The results of the new seismological observations point to an iron nucleus of the earth and therefore to a very great increase.

We may therefore probably assume that the difference in phase is very little, at any rate that difference in phase is slight for variations of short duration.

The magnetic force observed consisted of a primary and of a secondary part, which have the same sign as far as the horizontal component is concerned,

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[1]) Sitz. Ber. der Berliner Akademie für 1895.

The ratio c' between the secondary and primary part increases with the frequency of the vibrations of the current.

Let us call the magnetic force X, then

$$X = \text{primary} + \text{secondary}$$
$$X = \text{primary}\ (1 + c'),$$

so the primary part of $X_0 = \frac{X}{1 + c'}$.

The extra-terrestrial current which we can put $S = s \sin 2\pi \frac{t}{T}$ will induct in the upper earthstrata a current S':

$$S' = \varrho f(ST) \frac{2\pi}{T} \cos 2\pi \frac{t}{T}.$$

The induction will depend on the distance and the latter possibly on intensity and duration of the vibration of the current, moreover on the conductivity ϱ of the upper earthstrata.

The primary magnetic force will at first approximation (the distance being about the same) depend in the same manner on the extra-terrestrial current. So:

$$X_0 = f(sT) \sin 2\pi \frac{t}{T}$$

and

$$X = (1 + c') f(sT) \sin 2\pi \frac{t}{T}.$$

The existence of a vertical conducting current having been proved we must also take for granted that part of the extra-terrestrial current is closed by the earth and that a current is generated equally directed as the current of induction.

Already the properties of the conductivity of the atmosphere point to a dependence of magnitude and duration of the extra-terrestrial current, also on the conductivity of the upper earthstrata.

So we put for the current

$$\varrho\psi(sT) \sin 2\pi \frac{t}{T}.$$

And for the total current we find:

$$A = \varrho \left[\frac{2\pi}{T} f(sT) \cos 2\pi \frac{t}{T} + \psi(sT) \sin 2\pi \frac{t}{T} \right]$$

or

$$A = \varrho \left[\psi^2(sT) + \frac{4\pi^2}{T^2} f^2(sT) \right]^{1/2} \sin \frac{2\pi}{T} \left(k + \frac{T}{2\pi} Bg\ tg \frac{f(sT)}{\psi(sT)} \frac{?\pi}{T} \right),$$

whilst we found above

$$X = (1 + c') f(sT) \sin 2\pi \frac{t}{T}.$$

For the difference in phase we found a constant part of T or

$$\frac{T}{2\pi} Bg\, tg \frac{f(sT)}{\psi(sT)} \frac{2\pi}{T} = \frac{T}{K}.$$

For angles of $\pm 23°$, found for Cheribon for a shorter duration of the vibration, we may write here approximately

$$\frac{f(s\,T)}{\psi(s\,T)} = \frac{T}{K}.$$

So the ratio of amplitudes becomes

$$\frac{A}{X} = \frac{\varrho}{1 + c'} \frac{\psi(s\,T)}{f(s\,T)} \left[1 + \frac{4\pi^2}{K^2}\right]^{1/2},$$

$$\frac{A}{X} = \frac{\varrho}{1 + c'} \frac{1}{T} \frac{1}{\sqrt{K^2 + 4\pi^2}} = C \frac{\varrho}{1 + c'} \frac{1}{T}.$$

For $1 + c'$ we find according to LAMB-SCHUSTER for that part of the potential which is to be expanded in terms of a spherical function of order 2 (for that part which is to be expanded in terms of spherical functions of a higher order, the increase is quicker).

$\sigma = \frac{constant}{T}$	$1 + c'$
10	1.172
20	1.278
30	1.337
40	1.374
50	1.399
100	1.466
900	1.605
6400	1.643

So for higher frequency (for duration of half a vibration $= 1$ minute σ is 7200) $\frac{1}{1 + c'}$ is about constant.

So we get $\frac{A}{X}$ proportional to $\frac{1}{T}$.

The observations, however, give for Cheribon and Anjer proportionality with $\sqrt[8]{\frac{1}{T}}$ (for still smaller T, even inversion, and for Semarang and Soerabaja proportionality to $\sqrt[4]{\frac{1}{T}}$.

AD. SCHMIDT (Met. Zeitsch. 1902 p. 94) brings the supposition forward that the current can be generated in the wire by induction only, thereby supposing the wire to be closed by the earth.

Then putting the case very simply we arrive by application of the rule of AMPÈRE at :

$$A_{E-W} = -\gamma \frac{dX}{dt} + \delta \frac{d^2Z}{d\varphi\, dt}.$$

This gives the difference with respect to the above that the variation Z makes its appearance.

The Z, however, changes but little in equatorial regions, so it cannot make the theory correspond to the observations.

The slow increase of the earth-current when the frequency increases does *not* point to induction, but rather to direct connection with the primary current.

The quantity $\frac{1}{1+c'}$ has indeed, compared to $\frac{1}{T}$, rather a slow course.

That difference in phase is according to SCHUSTER-LAMB rather decreasing for quicker vibration whilst for the earth-current it proves to be constant.

But whence the difference in phase?

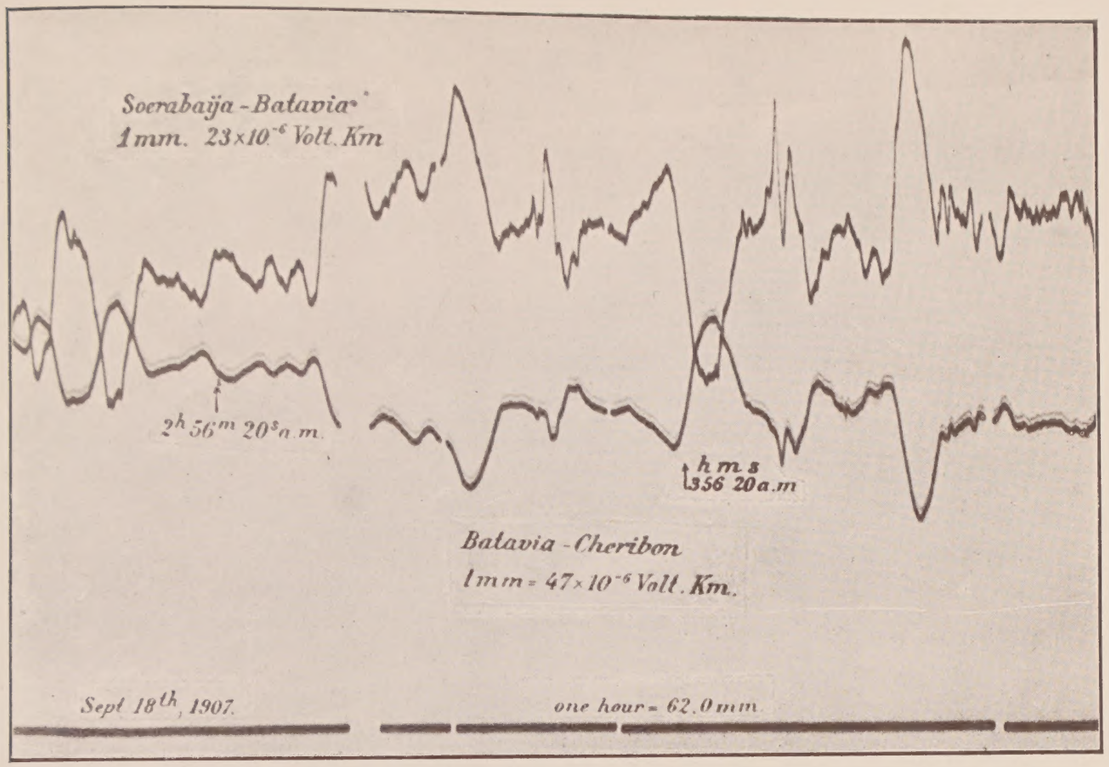
The differences in the intensity and the difference in phase of the earth-current for the lines to Anjer, Cheribon, Billiton and Buitenzorg can be explained also by the difference in conductivity of the ground between those places and Batavia.

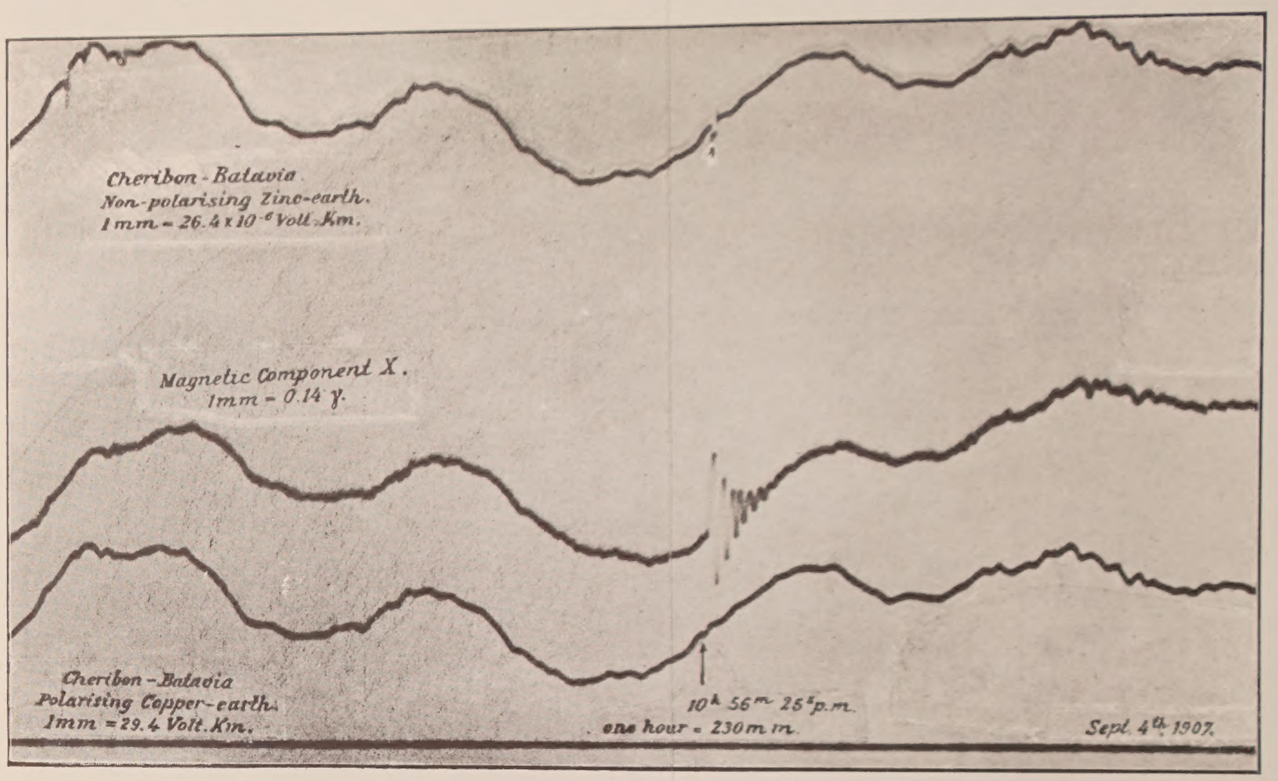
For instance between Anjer and Batavia lies the volcano Karang and therefore the conductivity is probably greater than between Batavia and Cheribon, and the fact that the earth-current is four times as strong can be attributed to it.

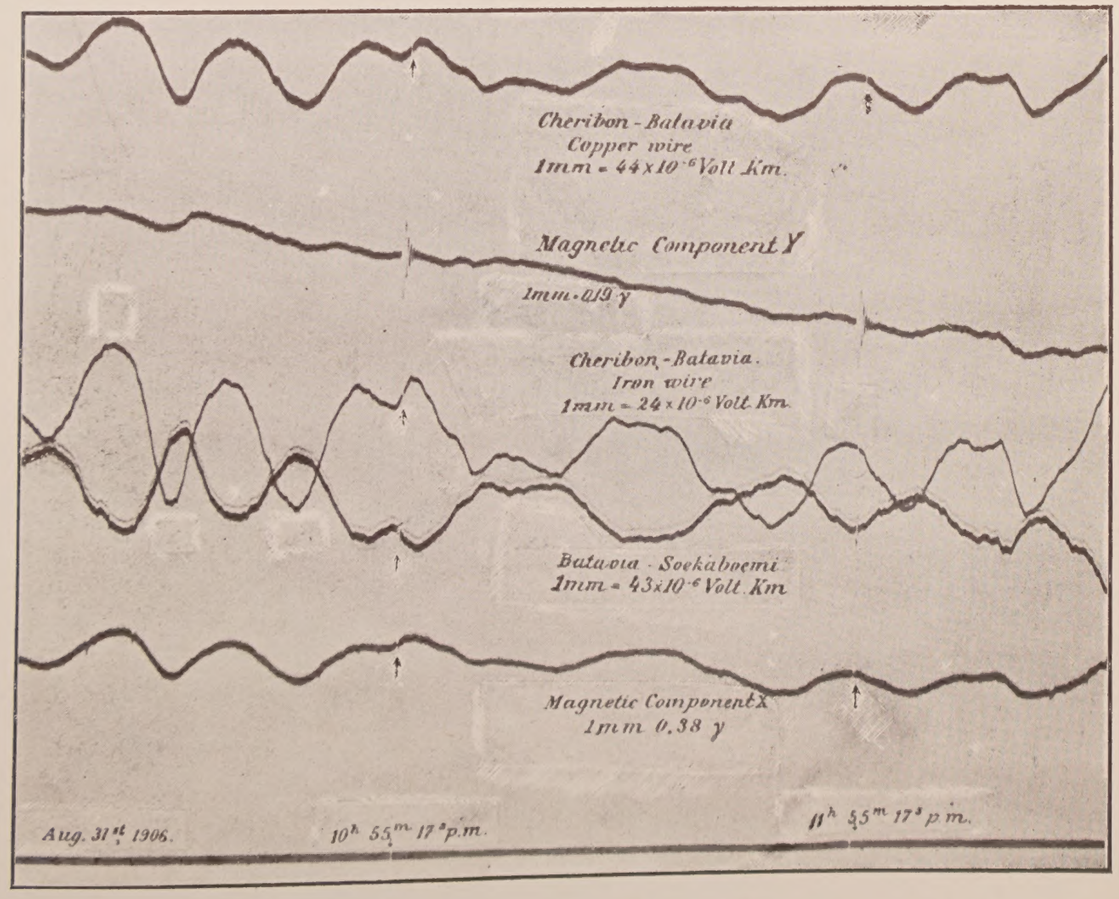
The great intensity of the earth current for Buitenzorg—Batavia may be partly attributed to the same reason and moreover to the difference in height (280 M); that between Batavia and Billiton to the well conducting seawater.

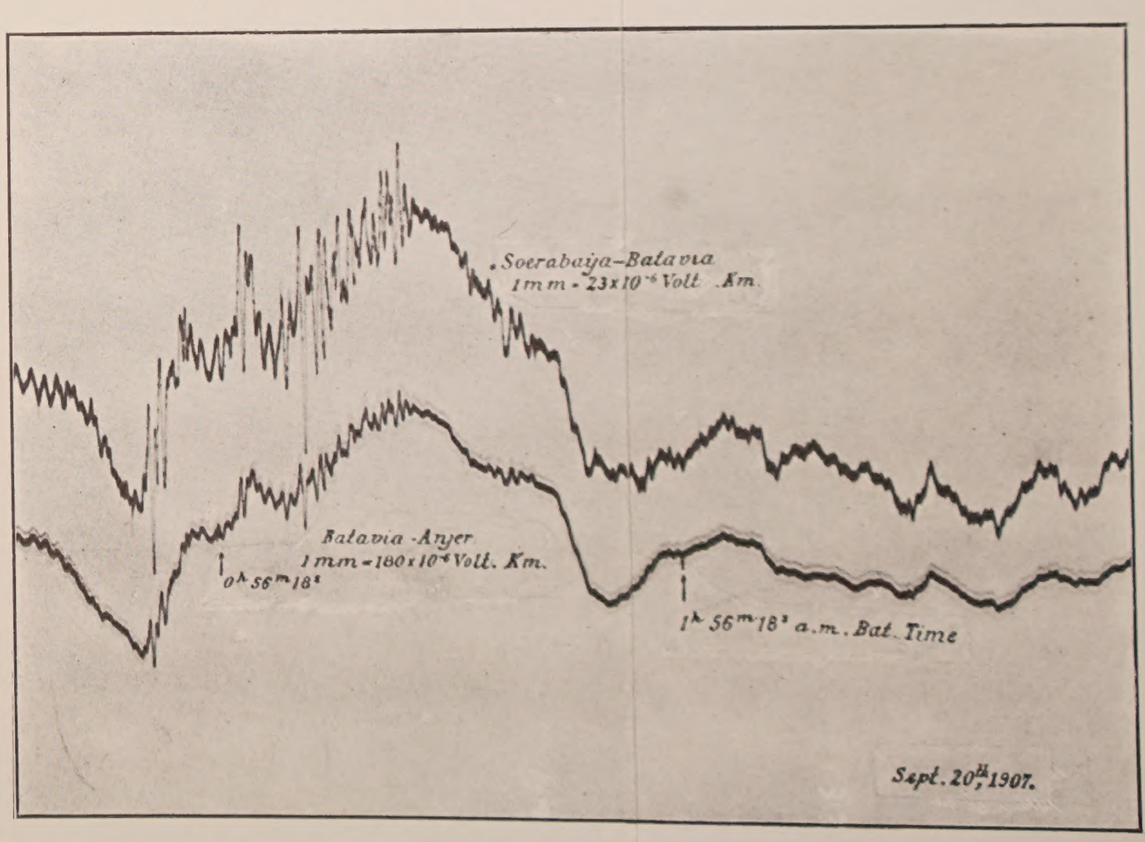
For the lines Semarang—Batavia and Soerabaja—Batavia we find however for the ratio of amplitude a distinct difference for vibrations of short duration. Each attempt at explanation of the connection between earth-current and magnetic variation will be in vain as long as this has not been confirmed and expounded.

To explain it out of the loss by isolation is impossible, as the difference would have to appear less for the lines Anjer-Batavia (106 K. M.) and Cheribon-Batavia (200 K. M.) which is not the case.


Soerabaija - Batavia
1 mm. 23×10⁻⁶ Volt. Km
2h 56m 20s a.m.
h m s
3 56 20 a.m
Batavia - Cheribon
1 mm = 47×10⁻⁶ Volt. Km.
Sept 18th, 1907.
one hour = 62.0 mm


Cheribon - Batavia
Non-polarising Zinc-earth.
1 mm = 26.4×10⁻⁶ Volt. Km.
Magnetic Component X.
1 mm = 0.14 γ.
Cheribon - Batavia
Polarising Copper-earth.
1 mm = 29.4 Volt. Km.
10h 56m 25s p.m.
one hour = 230 m m
Sept. 4th, 1907.


Cheribon - Batavia
Copper wire
1 mm = 44×10⁻⁶ Volt Km.
Magnetic Component Y
1 mm. 0.19 γ
Cheribon - Batavia.
Iron wire
1 mm = 24×10⁻⁶ Volt Km.
Batavia - Soekaboemi
1 mm = 43×10⁻⁶ Volt. Km
Magnetic Component X
1 m m 0.38 γ
Aug. 31st, 1906.
10h 55m 17s p.m.
11h 55m 17s p.m.


Soerabaija - Batavia
1 m m - 23×10⁻⁶ Volt Km.
Batavia - Anjer
1 mm = 180×10⁻⁶ Volt. Km.
0h 56m 18s
1h 56m 18s a.m. Bat. Time
Sept. 20th, 1907.

Neither can it be explained by mutual induction of the two lines, passing partly along the same telegraphpoles, as that influence would just work inversely.

There is a circumstance which causes the lines to Anjer and Cheribon to differ greatly from those to Semarang and Soerabaja; that is the greatest depth below the surface of the earth which the chord reaches between those places and Batavia.

It is for	Batavia—Anjer	1	K. M.
	Batavia—Cheribon	3	,,
	Batavia—Semarang	14	,,
	Batavia—Soerabaja	37	,,

When thus the variations of short duration cause a current chiefly at a greater depth, where the conductivity is very different from than at the surface, a distinct difference might appear. The opposite however is more to be expected.

To conform that difference it will be necessary to register at Semarang the current between Cheribon-Semarang and Soerabaja-Semarang.

If we find for that the same as for Batavia-Cheribon and Batavia-Anjer, then indeed we must attribute the greater increase of amplitude with short duration for the lines Batavia-Semarang and Batavia-Soerabaja to the greater distance.

ERRATUM.

In the Proceedings of the Meeting of March 30, 1907:

p. 770 l. 3 from the bottom: for 46.419 read 46.491.
p. 779 l. 10 from the top: for VII H_2 1 read VII H_2 2.

(February 20, 1908).